MICROECONOMICS 2

LECTURE 3
Demand

I have enough money to last me the rest of my life, unless I buy something.

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Outline

Challenge

Paying Employees to Relocate

1. **Deriving Demand Curves**
2. **Effects of an Increase in Income**
3. **Effects of a Price Increase**
4. **Cost-of-Living Adjustment**
5. **Revealed Preference**

Challenge Solution

Challenge:
Paying Employees to Relocate

Background

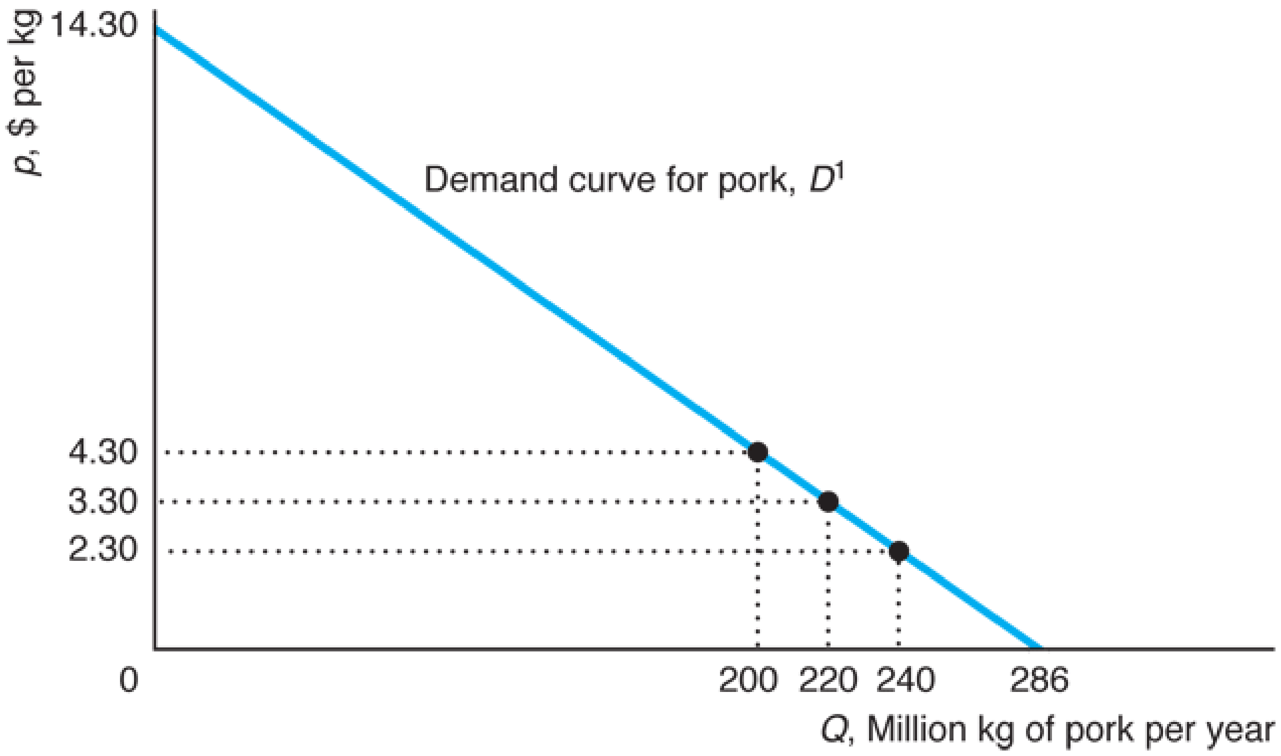
- International firms are increasingly relocating workers throughout their home countries and internationally.
- Firms must decide how much compensation to offer workers to move.

Questions

- Do firms’ standard compensation packages overcompensate workers by paying them more than necessary to include them to move to a new location?

1. Deriving Demand Curves

- If we hold people's tastes, their incomes, and the prices of other goods constant, a change in the price of a good will cause a ***movement along the demand curve***.
- We saw this in Lecture 1:



Deriving Demand Curves

- In Lecture 2, we used calculus to maximize consumer utility subject to a budget constraint.
  - This amounts to solving for the consumer's system of demand functions for the goods.
- Example: q_1 = pizza and q_2 = burritos
  - Demand functions express these quantities in terms of the prices of both goods and income:

$$q_1 = Z(p_1, p_2, Y)$$

$$q_2 = B(p_1, p_2, Y)$$

  - Given a specific utility function, we can find closed-form solutions for the demand functions.

Example: Deriving Demand Curves

- Constant Elasticity of Substitution (CES) utility function:

$$U(q_1, q_2) = \left(q_1^{\rho} + q_2^{\rho}\right)^{\frac{1}{\rho}},\ 0 \neq \rho \leq 1$$

- Budget constraint: Y= p1q1 + p2q2
- In Chapter 3, we learned that the demand functions that result from this constrained optimization problem are:

$$q_1 = \frac{Yp_1^{z-1}}{p_1^z + p_2^z} \qquad q_2 = \frac{Yp_2^{z-1}}{p_1^z + p_2^z}$$

- Quantity demanded of each good is a function of the prices of both goods and income.

Example: Deriving Demand Curves

- Cobb-Douglas utility function: $U(q_1, q_2) = q_1^a q_2^{1-a}$
- Budget constraint: $Y = p_1q_1 + p_2q_2$
- In Lecture 2, we learned that the demand functions that result from this constrained optimization problem are:

$$q_1 = a\frac{Y}{p_1} \qquad q_2 = (1-a)\frac{Y}{p_2}$$

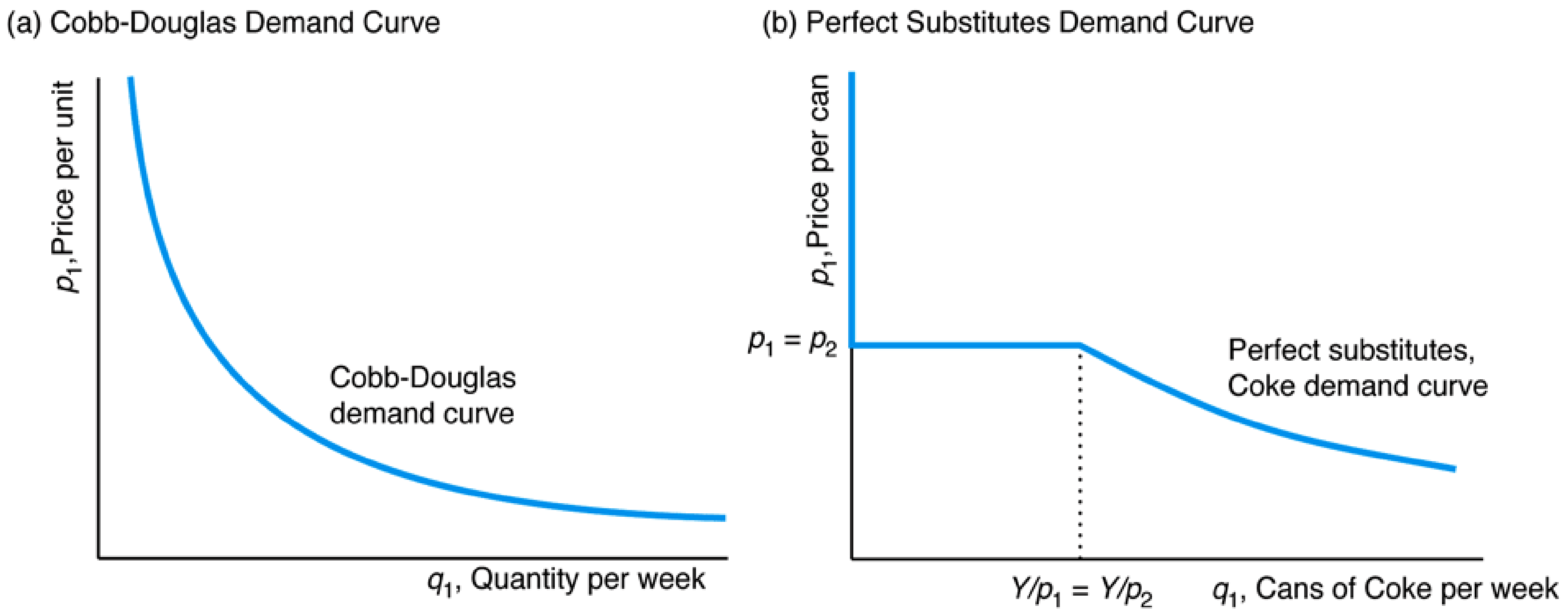
- With Cobb-Douglas, quantity demanded of each good is a function of only the good's own-price and income.

Demand Functions for Five Utility Functions

Utility Function	$U(q_1, q_2)$	Solution	Demand Functions: q_1	Demand Functions: q_2
Perfect complements	$\min(q_1, q_2)$	interior	$Y/(p_1 + p_2)$	$Y/(p_1 + p_2)$
CES, $\rho \neq 0, \rho < 1, \sigma = 1/(\rho - 1)$	$(q_1^\rho + q_2^\rho)^{\frac{1}{\rho}}$	interior	$q_1 = \dfrac{Yp_1^\sigma}{p_1^{\sigma+1} + p_2^{\sigma+1}}$	$q_2 = \dfrac{Yp_2^\sigma}{p_1^{\sigma+1} + p_2^{\sigma+1}}$
Cobb-Douglas	$q_1^a q_2^{1-a}$	interior	aY/p_1	$(1 - a)Y/p_2$
Perfect substitutes, $p_1 = p_2 = p$	$q_1 + q_2$	interior	$q_1 + q_2 = Y/p$	
$p_1 < p_2$		corner	Y/p_1	0
$p_1 > p_2$		corner	0	Y/p_2
Quasilinear,	$aq_1^{0.5} + q_2$			
$Y > a^2p_2/[4p_1]$		interior	$\left(\dfrac{a}{2}\dfrac{p_2}{p_1}\right)^2$	$\dfrac{Y}{p_2} - \dfrac{a^2}{4}\dfrac{p_2}{p_1}$
$Y \leq a^2p_2/[4p_1]$		corner	Y/p_1	0

Deriving Demand Curves

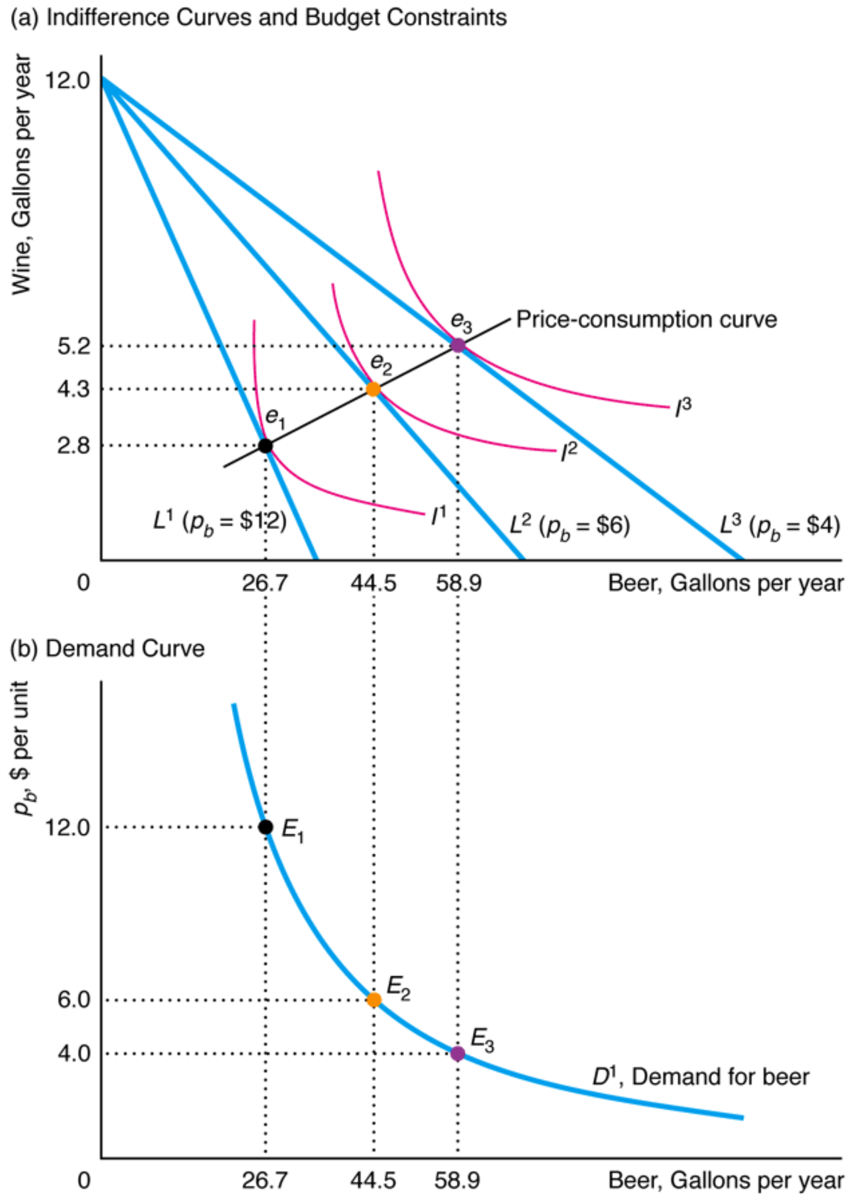
Panel (a) below shows the demand curve for q_1, which we plot by holding Y fixed and varying p_1.



Deriving Demand Curves Graphically

Allowing the price of the good on the x-axis to fall, the budget constraint rotates out and shows how the optimal quantity of the x-axis good purchased increases.

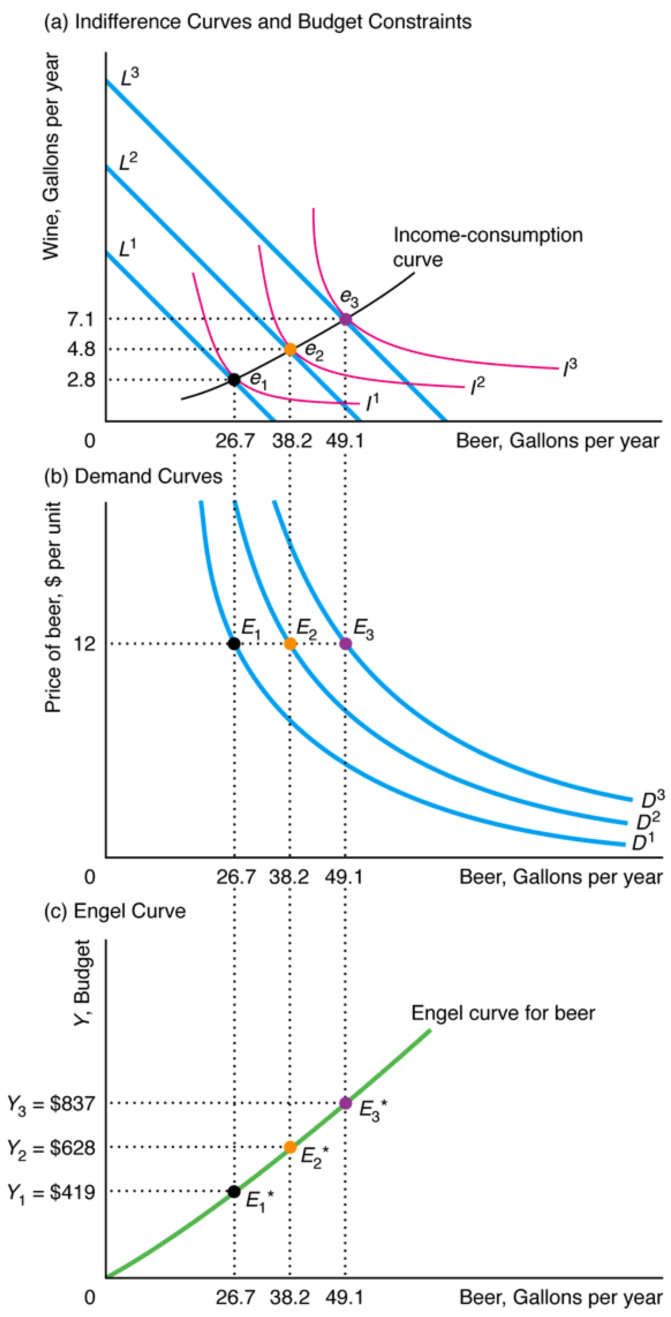
- This traces out points along the demand curve.



2. Effects of an Increase in Income

- An increase in an individual's income, holding tastes and prices constant, causes a ***shift of the demand curve***.
    - An increase in income causes an increase in demand (e.g., a parallel shift away from the origin) if the good is a ***normal good*** and a decrease in demand (e.g., parallel shift toward the origin) if the good is ***inferior***.
- A change in income prompts the consumer to choose a new optimal bundle.
- The result of the change in income and the new utility maximizing choice can be depicted three different ways.

Effects of a Budget Increase



Effects of an Increase in Income

The result of the change in income and the new utility maximizing choice can be depicted three different ways.

1. **Income-consumption curve**: using the consumer utility maximization diagram, traces out a line connecting optimal consumption bundles.
2. **Shifts in demand curve**: using demand diagram, show how quantity demanded increases as the price of the good stays constant.
3. **Engle curve**: with income on the vertical axis, show the positive relationship between income and quantity demanded.

Consumer Theory and Income Elasticities

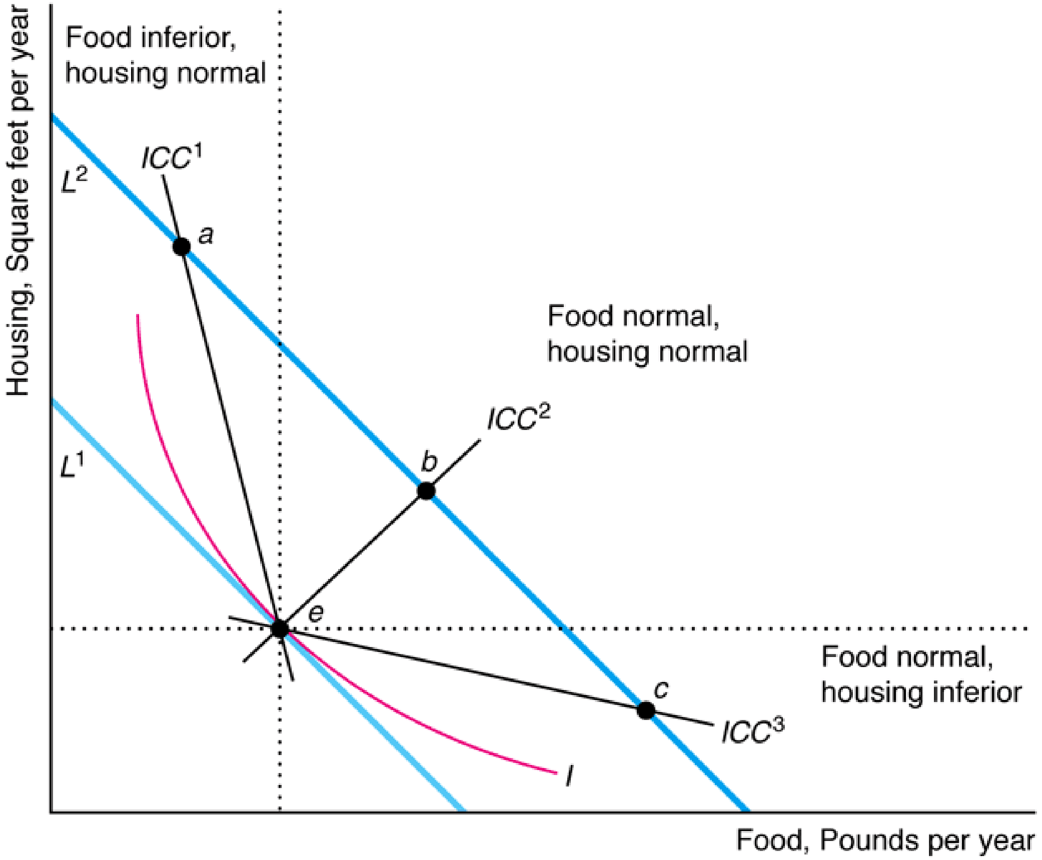
- Recall the formula for income elasticity of demand from Lecture 1:

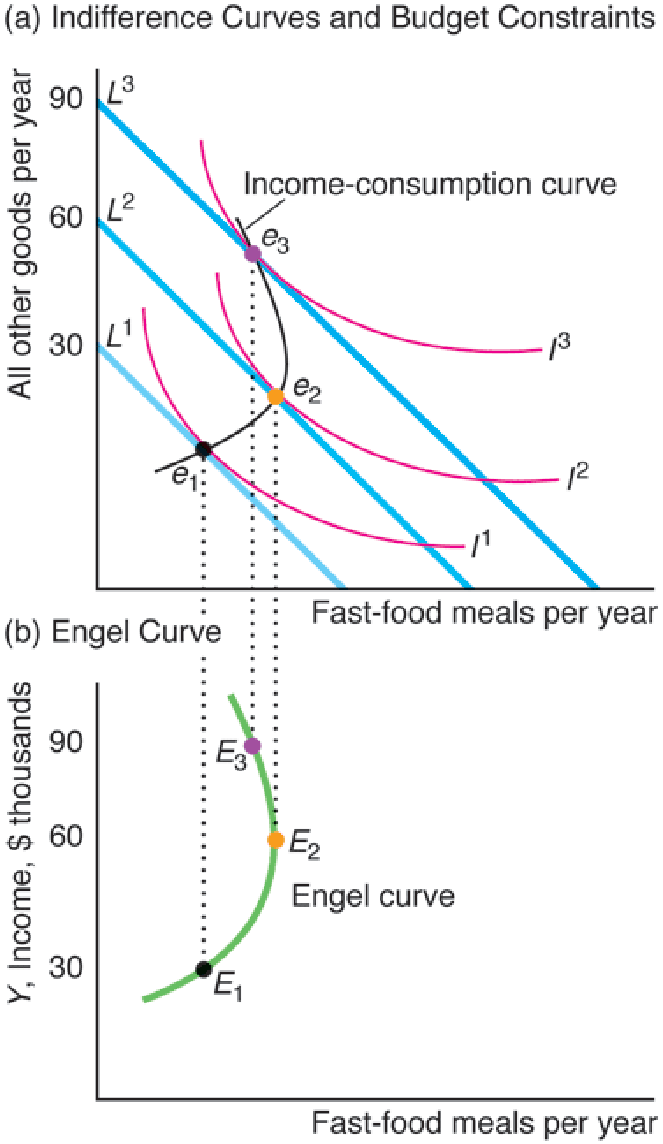
$$\xi = \frac{\text{percentage change in quantity demanded}}{\text{percentage change in income}} = \frac{\Delta Q/Q}{\Delta Y/Y} = \frac{\partial Q}{\partial Y}\frac{Y}{Q}$$

- ***Normal goods***, those goods that we buy more of when our income increases, have a positive income elasticity.
  - ***Luxury goods*** are normal goods with an income elasticity greater than 1.
  - ***Necessity goods*** are normal goods with an income elasticity between 0 and 1.
- ***Inferior goods***, those goods that we buy less of when our income increases, have a negative income elasticity.

Income-Consumption Curve and Income Elasticities

The shape of the income-consumption curve for two goods tells us the sign of their income elasticities.





Income-Consumption Curve and Income Elasticities

The shape of the income-consumption and Engle curves can change in ways that indicate goods can be both normal and inferior, depending on an individual's income level.

3. Effects of a Price Increase

- Holding tastes, other prices, and income constant, an increase in the price of a good has two effects on an individual's demand:

    1. **Substitution effect**: the change in quantity demanded when the good's price increases, holding other prices and consumer utility constant.

    2. **Income effect**: the change in quantity demanded when income changes, holding prices constant.

- When the price of a good increases, the total change in quantity demanded is the sum of the substitution and income effects.

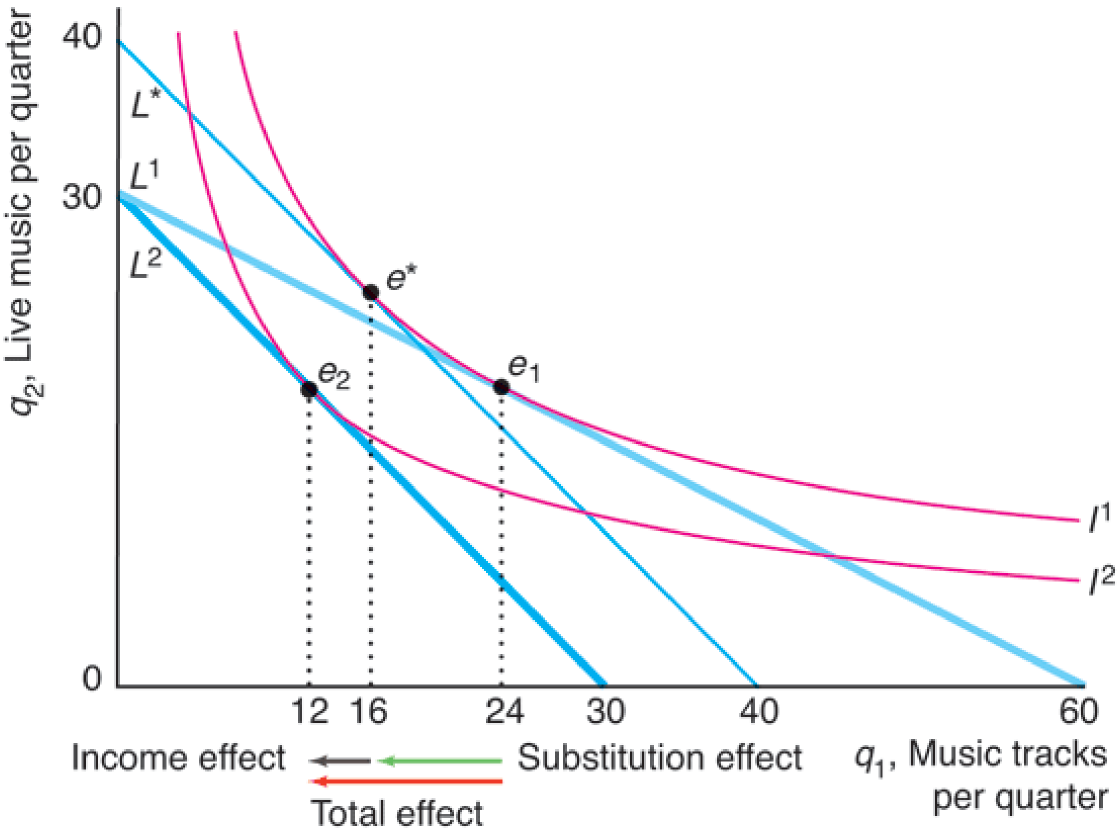
Income and Substitution Effects

- The direction of the substitution effect is always negative.
  - When price increases, individuals consume less of it because they are substituting away from the now more expensive good.
- The direction of the income effect depends upon whether the good is normal or inferior; it depends upon the income elasticity.
  - When price increases and the good is normal, the income effect is negative.
  - When price increases and the good is inferior, the income effect is positive.

Income and Substitution Effects with a Normal Good

Beginning from budget constraint L^1, an increase in the price of music tracks rotates budget constraint into L^2.

- The total effect of this price change, a decrease in quantity of 12 tracks per quarter, can be decomposed into income and substitution effects.

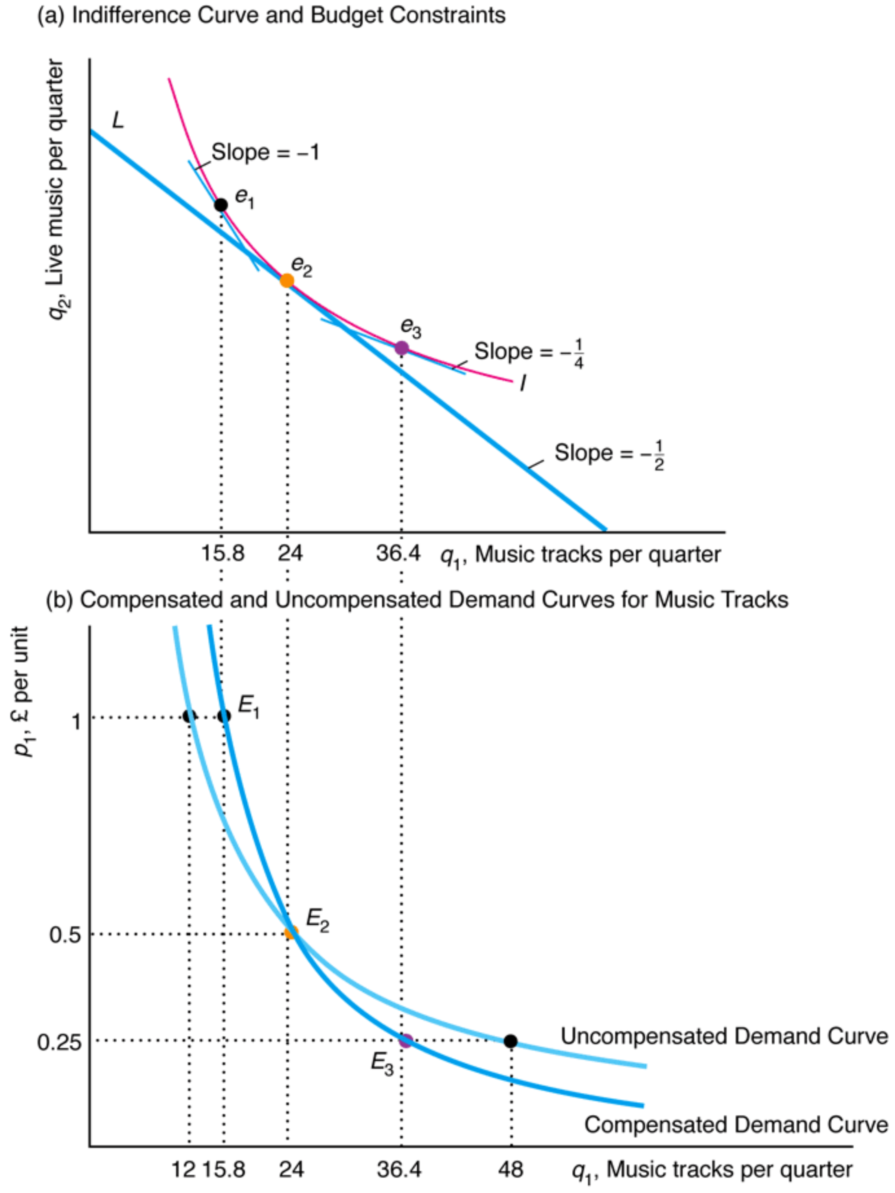


Compensated Demand Curve

- The demand curves shown thus far have all been ***uncompensated***, or ***Marshallian***, ***demand curves***.
  - Consumer utility is allowed to vary with the price of the good.
  - In the figure from the previous slide, utility fell when the price of music tracks rose.
- Alternatively, a ***compensated***, or ***Hicksian***, ***demand curve*** shows how quantity demanded changes when price increases, holding utility constant.
  - Only the pure substitution effect of the price change is represented in this case.
  - An individual must be compensated with extra income as the price rises in order to hold utility constant.

Compensated Demand Curve

- In calculating compensated demand curve for music tracks, vary the price of music tracks, compensate income to hold utility constant.
- Determine the quantity demanded



Compensated Demand Curve

- Deriving the compensated, or Hicksian, demand curve is straight-forward with the expenditure function:
  - *E* is the smallest expenditure that allows the consumer to achieve a given level of utility based on given market prices: $E = E\left(p_1, p_2, \overline{U}\right)$
  - Differentiating with respect to the price of the first good yields the compensated demand function for the first good:

$$\frac{\partial E}{\partial p_1} = H\left(p_1,\ p_2,\ \overline{U}\right) = q_1$$

✓ A \$1 increase in p_1 on each of the q_1 units purchased requires the consumer increases spending by \$$q_1$ to keep utility constant.

✓ This result is called Shephard's lemma.

Slutsky Equation

- We graphically decomposed the total effect of a price change on quantity demanded into income and substitution effects.
- Deriving this same relationship mathematically utilizes elasticities and is called the ***Slutsky equation***.

$$\begin{array}{ccccc} \text{total effect} & = & \text{substitution effect} & + & \text{income effect} \\ \varepsilon & = & \varepsilon^{*} & + & (-\theta\xi) \end{array}$$

- ε is elasticity of uncompensated demand and the total effect
- ε^{*} is elasticity of compensated demand and the substitution effect
- θ is the share of the budget spent on the good
- ξ is the income elasticity
- $\theta\xi$ is the income effect

4. Cost-of-Living Adjustment

- ***Consumer Price Index (CPI)***: measure of the cost of a standard bundle of goods (market basket) to compare prices over time.
  - Example: In 2012 dollars, what is the cost of a McDonald's hamburger in 1955?

$$\frac{\text{CPI for 2012}}{\text{CPI for 1955}} \times \text{price of a burger} = \frac{229.1}{26.7} \times 15¢ = \$1.29.$$

- Knowledge of substitution and income effects allows us to analyze how accurately the government measures inflation.
- Consumer theory can be used to show that the cost-of-living measure used by governments overstates inflation.

Cost-of-Living Adjustment (COLA)

- CPI in first year is the cost of buying the market basket of food (F) and clothing (C) that was actually purchased that year:

$$Y_1 = p_C^1 C_1 + p_F^1 F_1$$

- CPI in the second year is the cost of buying the first year's bundle in the second year:

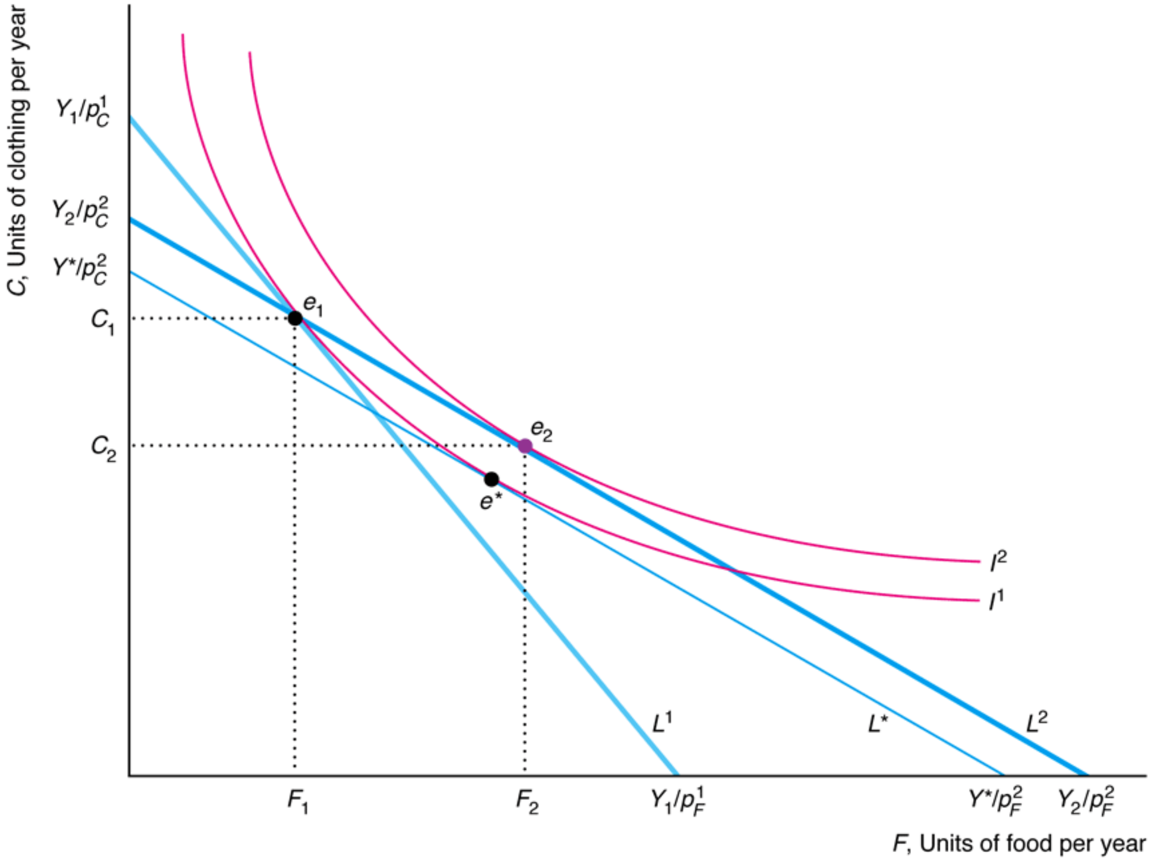
$$Y_2 = p_C^2 C_1 + p_F^2 F_1$$

- The rate of inflation determines how much additional income it took to buy the first year's bundle in the second year:

$$\frac{Y_2}{Y_1} = \frac{p_C^2 C_1 + p_F^2 F_1}{p_C^1 C_1 + p_F^1 F_1}$$

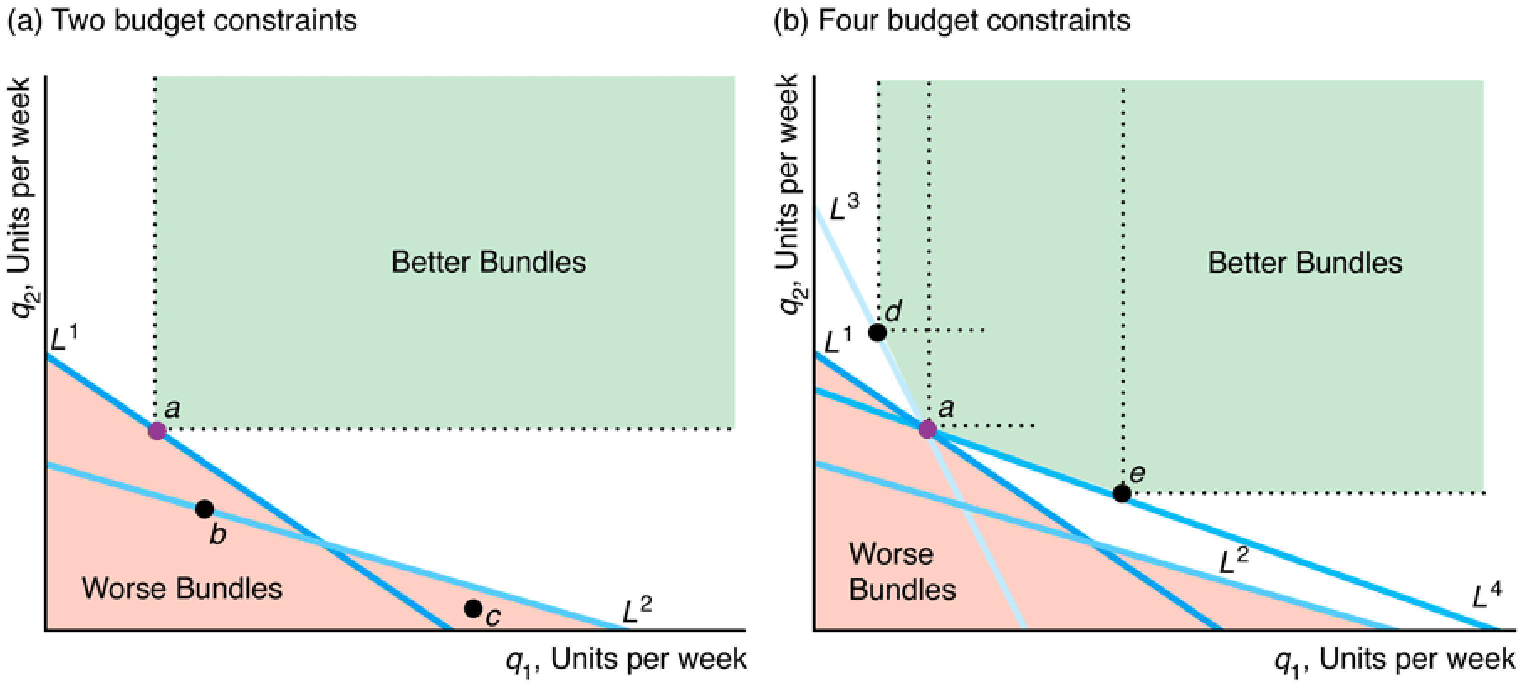
Cost-of-Living Adjustment (COLA)

- If a person's income increases automatically with the CPI, he can afford to buy the first year's bundle in the second year but chooses not to.
  - Better off in the second year because the CPI-based COLA overcompensates in the sense that utility increases.



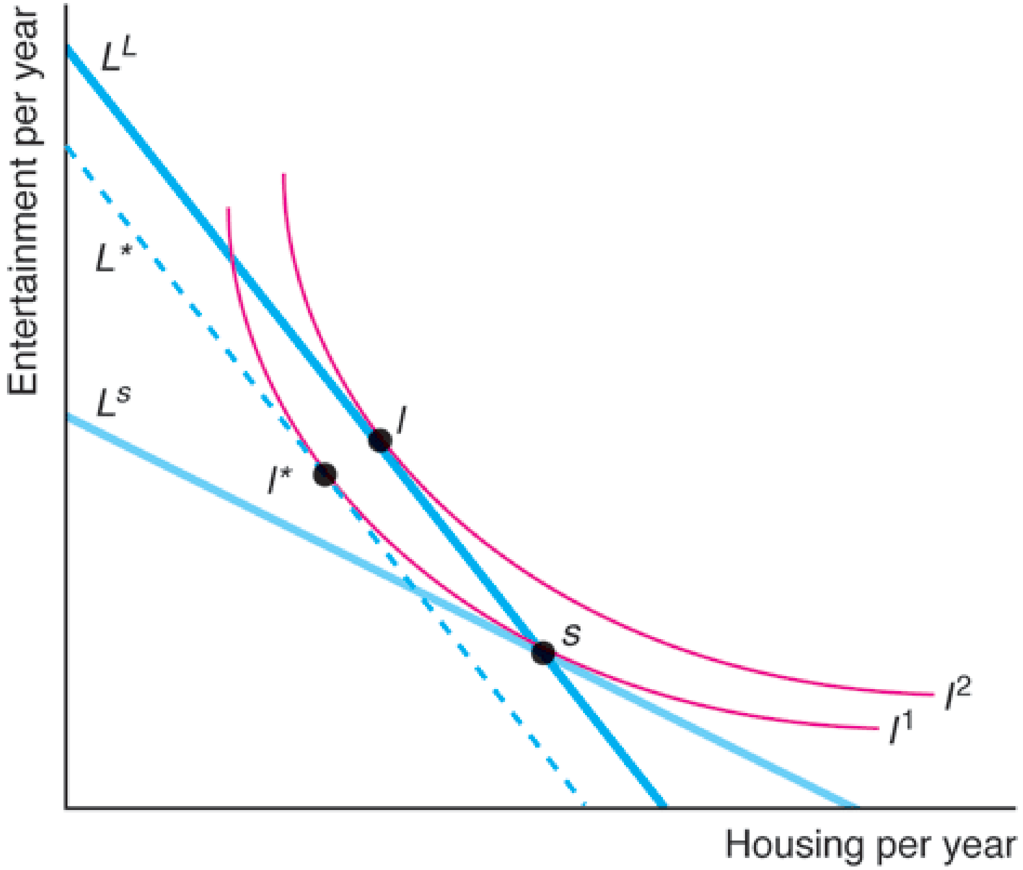
5. Revealed Preference

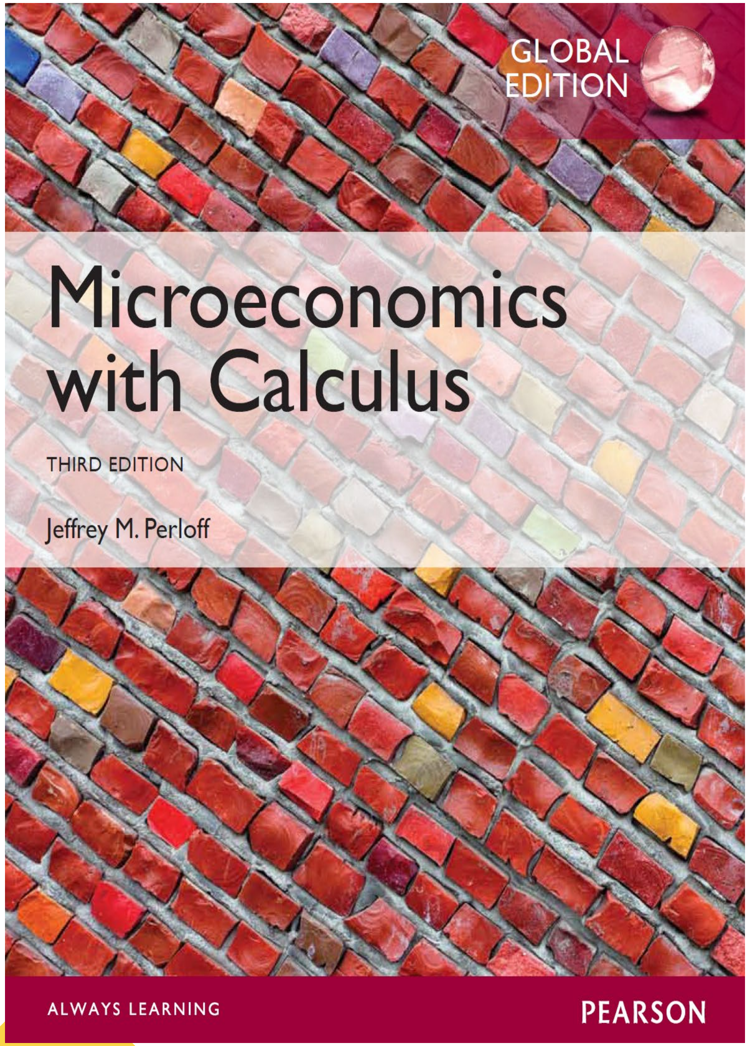
- Preferences → predict consumer's purchasing behavior
- Purchasing behavior → infer consumer's preferences



Challenge Solution

- Relocate from Seattle to London
- Budget line in Seattle is L^s and buys s. Utility is I^1.
- Housing is relatively more expensive in London.
- If worker is compensated when moving to afford s in London, budget line is L^L. Worker consumes l and utility is I^2.
- Firm should compensate L^*. worker consumes l^* and utility is I^1.





REFERENCE

Chapter 4 - Microeconomics: Theory and Applications with Calculus, 3rd Edition. By Jeffrey M. Perloff. 2014 Pearson Education.