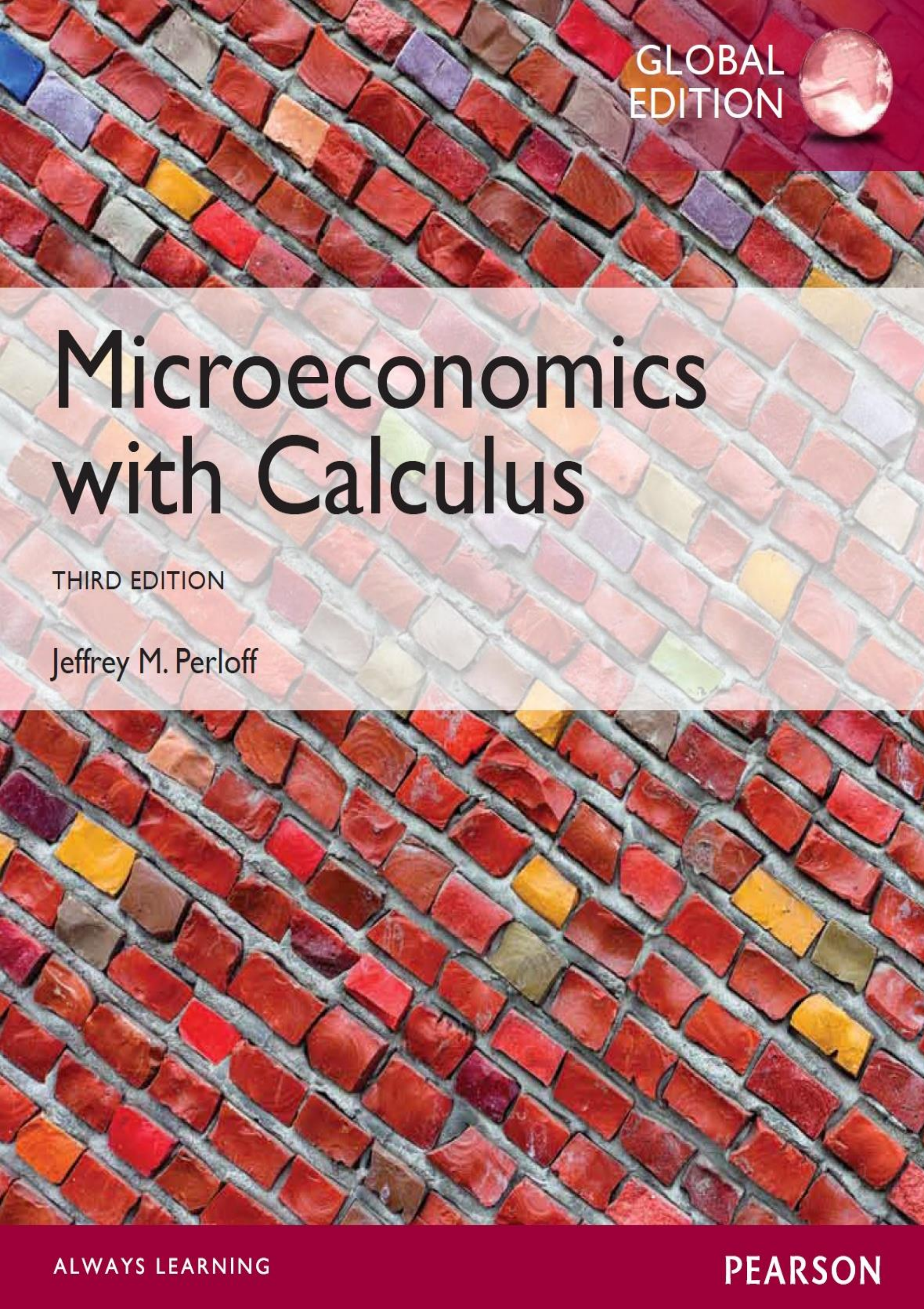


Lecture 1:

Decision under Risk

PEARSON

Outline

- Decisions under risk: Deciding whether to go to school or work, thinking about which job offer to accept, where to buy your house, what to invest your retirement savings in…
- Dual approach to analyzing risk and behavior
  - First, quantify risk statistically and mathematically
    - Probability, Variance, Expected Value
  - Next, model human behavior over risky scenarios
    - Expected Utility Theory
    - Non-Expected Utility Theory

1 Assessing Risk

2 Attitudes Toward Risk

3 Reducing Risk

4 Behavioral Economics on Decisions under Risk

Readings: Perloff 16.1-16.3, 16.5

1 Assessing Risk

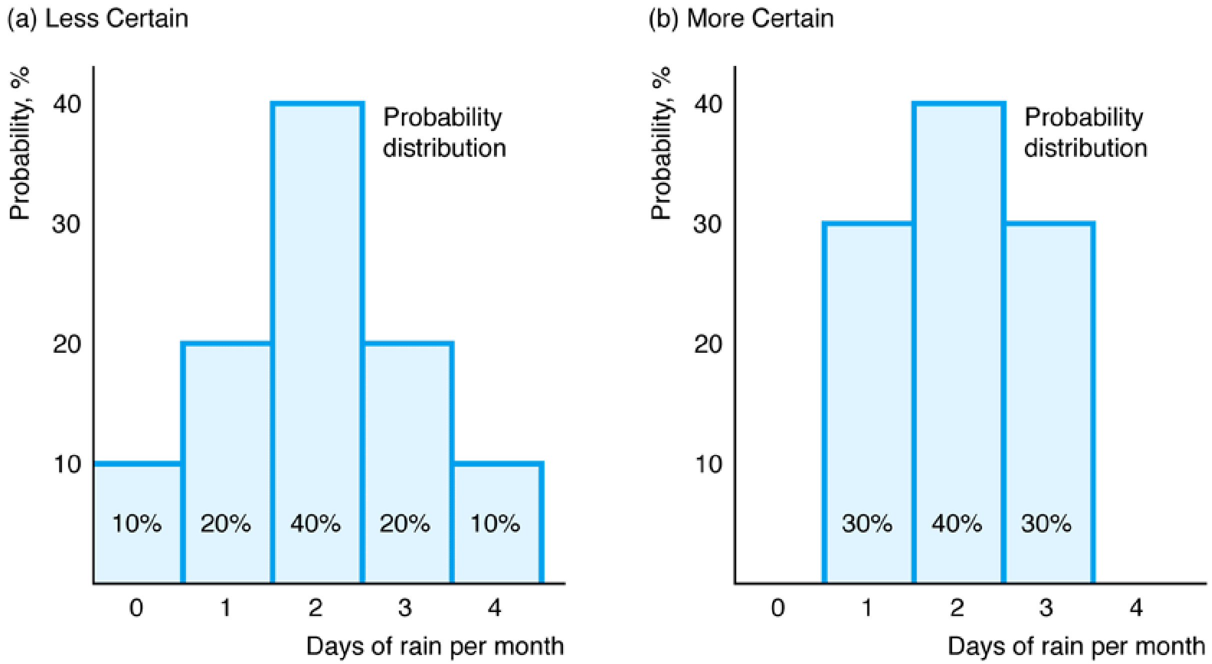
- ***Risk*** is the situation when the likelihood of each possible outcome is known or can be estimated, and no single possible outcome is certain to occur.
  - Ambiguity: You don't know the likelihoods; Unawareness: You don't know some possible outcomes
- A ***probability*** is a number between 0 and 1 that indicates the likelihood that a particular outcome will occur.
- We can estimate probability with ***frequency***, the number of times that one particular outcome of an event occurred (n) out of the total number of times that the event occurred (N).

$$\theta = \frac{n}{N}$$

- If we don't have a history of the event that allows us to calculate frequency, we can use our best estimate or ***subjective probability***.

1 Assessing Risk

- A ***probability distribution*** relates the probability of occurrence to each possible outcome.



1 Assessing Risk

- ***Expected value*** is the value of each possible outcome (V_i) times the probability of that outcome (θ_i), summed over all *n* possible outcomes:

$$\mathrm{EV} = \sum_{i=1}^{n} \theta_i V_i$$

- ***Variance*** measures the spread of the probability distribution or how much variation there is between the actual value and the expected value.

$$\text{Variance} = \sum_{i=1}^{n} \theta_i \left(V_i - \mathrm{EV}\right)^2$$

- ***Standard deviation*** (σ) is the square root of the variance and is a more commonly reported measure of risk.
  - Used by economists and business people to describe risk.

1 Assessing Risk

- Example: Greg schedules an <u>outdoor</u> event
  - If it doesn't rain, he'll make $15 in profit
  - If it does rain, he'll make -$5 in profit (loss)
  - There is a 50% chance of rain.
- Greg's expected value (outdoor event):

$$EV = [Pr(\text{no rain}) \times \text{Value}(\text{no rain})] + [Pr(\text{rain}) \times \text{Value}(\text{rain})]$$
$$= \left(\tfrac{1}{2} \times \$15\right) + \left[\tfrac{1}{2} \times (-\$5)\right] = \$5$$

- Variance (outdoor event):

$$\sigma^2 = \left[\theta_1 \times (V_1 - EV)^2\right] + \left[\theta_2 \times (V_2 - EV)^2\right]$$
$$= \left[\tfrac{1}{2} \times (\$15 - \$5)^2\right] + \left[\tfrac{1}{2} \times (-\$5 - \$5)^2\right]$$
$$= \left[\tfrac{1}{2} \times (\$10)^2\right] + \left[\tfrac{1}{2} \times (-\$10)^2\right] = \$100.$$

  - Standard deviation = $10

1 Assessing Risk

- Example, continued: Greg schedules an indoor event
  - If it doesn't rain, he'll make \$10 in profit
  - If it does rain, he'll make \$0 in profit
  - There is still a 50% chance of rain.
- Greg's expected value (indoor event)… is the same!

$$EV = \left(\tfrac{1}{2} \times \$10\right) + \left(\tfrac{1}{2} \times \$0\right) = \$5$$

- Variance (indoor event)… is much smaller:

$$\begin{aligned}\sigma^2 &= \left[\tfrac{1}{2} \times \left(\$10 - \$5\right)^2\right] + \left[\tfrac{1}{2} \times \left(\$0 - \$5\right)^2\right] \\ &= \left[\tfrac{1}{2} \times \left(\$5\right)^2\right] + \left[\tfrac{1}{2} \times \left(-\$5\right)^2\right] = \$25\end{aligned}$$

  - Standard deviation = \$5
  - Much less risky to schedule the event indoors.

2 Attitudes Toward Risk

- Although indoor and outdoor events have the same expected value, the outdoor event involves more risk.
  - He'll schedule the event outdoors only if he likes gambling
- People can be classified according to attitudes toward risk.
- A ***fair bet*** is a wager with an expected value of zero.
  - Example: You receive $1 if a flipped coin comes up heads and you pay $1 if a flipped coin comes up tails.
  - Someone who is unwilling to make a fair bet is ***risk averse***.
  - Someone who is indifferent about a fair bet is ***risk neutral***.
  - Someone who is ***risk preferring*** *(risk loving)* will make a fair bet.
- Implication: For any lottery, a risk averse person prefers taking EV(the lottery) to taking the lottery itself.
  - A lottery = EV (the lottery) + A fair bet
  - Vice versa for risk loving people.

2 Expected Utility Theory

- We can extend our model of utility maximization to include risk by assuming that people maximize *expected utility*.
- Expected utility, *EU*, is the probability-weighted average of the utility, $U(\bullet)$ from each possible outcome:

$$\mathrm{E}U = \sum_{i=1}^{n} \theta_i U\left(V_i\right)$$

  - The weights are the probabilities that each outcome will occur, just as in expected value.
- EU: the probability-weighted average of the utility from the outcomes
- EV: the probability-weighted average of the outcomes

2 Expected Utility Theory

- von Neumann and Morgenstern (1944) prove that a consumer whose preference satisfies completeness, transitivity, independence and continuity over any lotteries is an expected utility maximizer.
- Completeness and transitivity are defined as in Ch. 3.
- Independence: For any lotteries A, B, C, if A $\succsim$ B, then for any $0 \le t \le 1$, we have tA+(1-t)C $\succsim$ tB+(1-t)C.

Note: tA+(1-t)C means with prob. *t* you get lottery A, with prob. *1-t* you get lottery C.

- Continuity: For any lotteries A, B, C, if A $\succsim$ B $\succsim$ C, then there exists a probability *p* such that B ~ pA+(1-p)C.

Review: Consumer Theory

- To explain consumer behavior, economists assume that consumers have a set of tastes or preferences that they use to guide them in choosing between goods.
- Goods are ranked according to how much pleasure a consumer gets from consuming each.
  - Preference relations summarize a consumer's ranking
  - $\succ$ is used to convey strict preference (e.g. $a \succ b$)
  - $\succsim$ is used to convey weak preference (e.g. $a \succsim b$)
  - $\sim$ is used to convey indifference (e.g. $a \sim b$)

Review: Consumer Theory (cont.)

1. Completeness

- When facing a choice between a and b, a consumer can rank them so that either $a \succ b$, $b \succ a$, or $a \sim b$.
- Completeness also holds for weak preference:
- For any a and b, either $a \succsim b$ or $b \succsim a$.

2. Transitivity

- Consumers' rankings are logically consistent in the sense that if $a \succ b$ and $b \succ c$, then $a \succ c$.
- Transitivity also holds for weak preference and indifference.

Rationality=Completeness+Transitivity

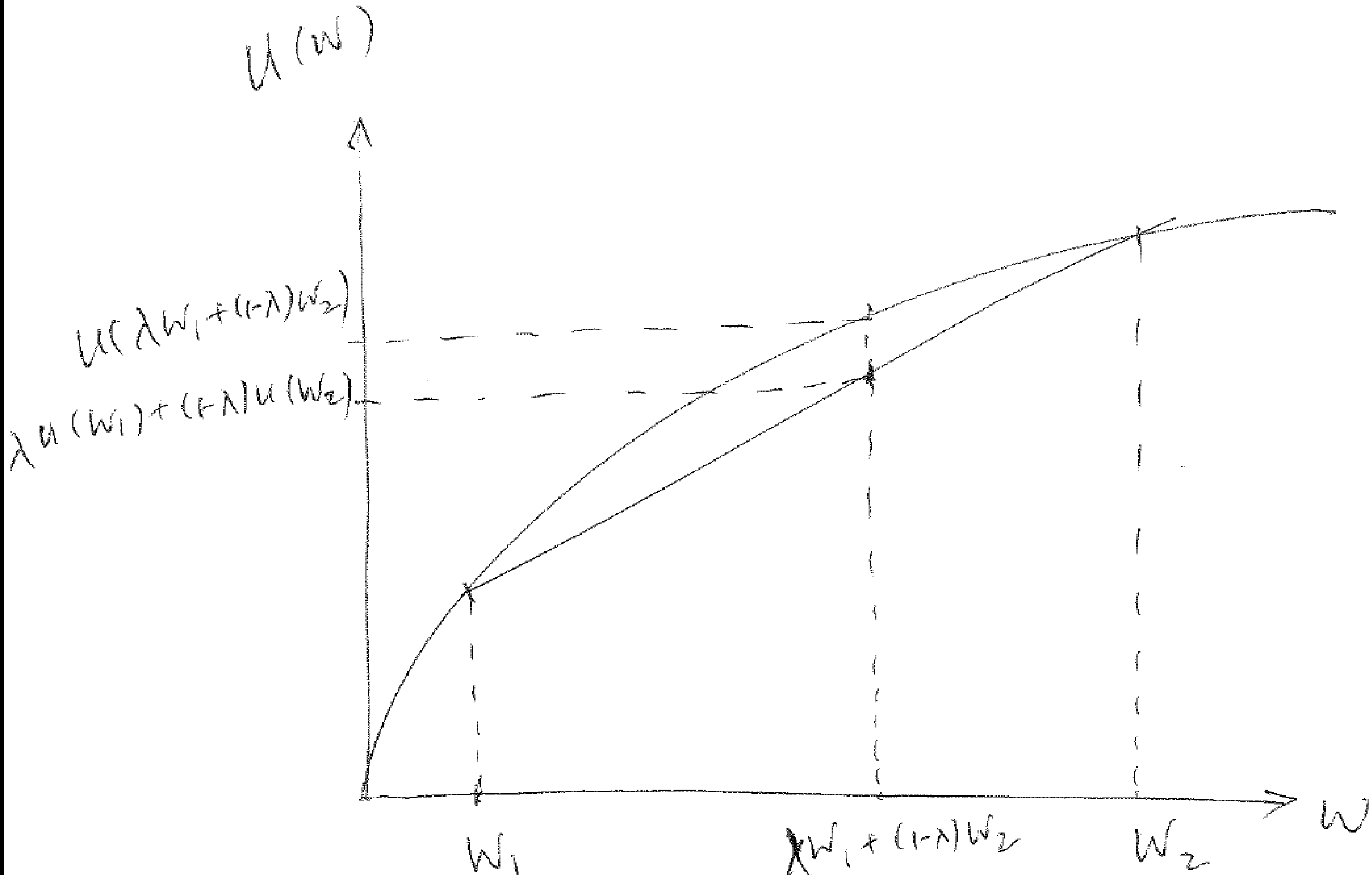
2 Expected Utility Theory

- With expected utility, a person whose utility function of income, *U(Y)*, is strictly concave ($U''<0$) is risk averse:
- Why strict concavity implies risk aversion? Mathematics
- For a strictly concave function U(.), we have: for any W_1, W_2, and any $0 < \lambda < 1$

$$U(\lambda W_1 + (1-\lambda)W_2) > \lambda U(W_1) + (1-\lambda)U(W_2)$$

- This means that for a gamble that gives you W_1 with prob. λ and W_2 with prob. $1-\lambda$, you prefer receiving EV to taking the gamble.

2 Expected Utility Theory



2 Expected Utility Theory

- Why strict concavity implies risk aversion? Intuition

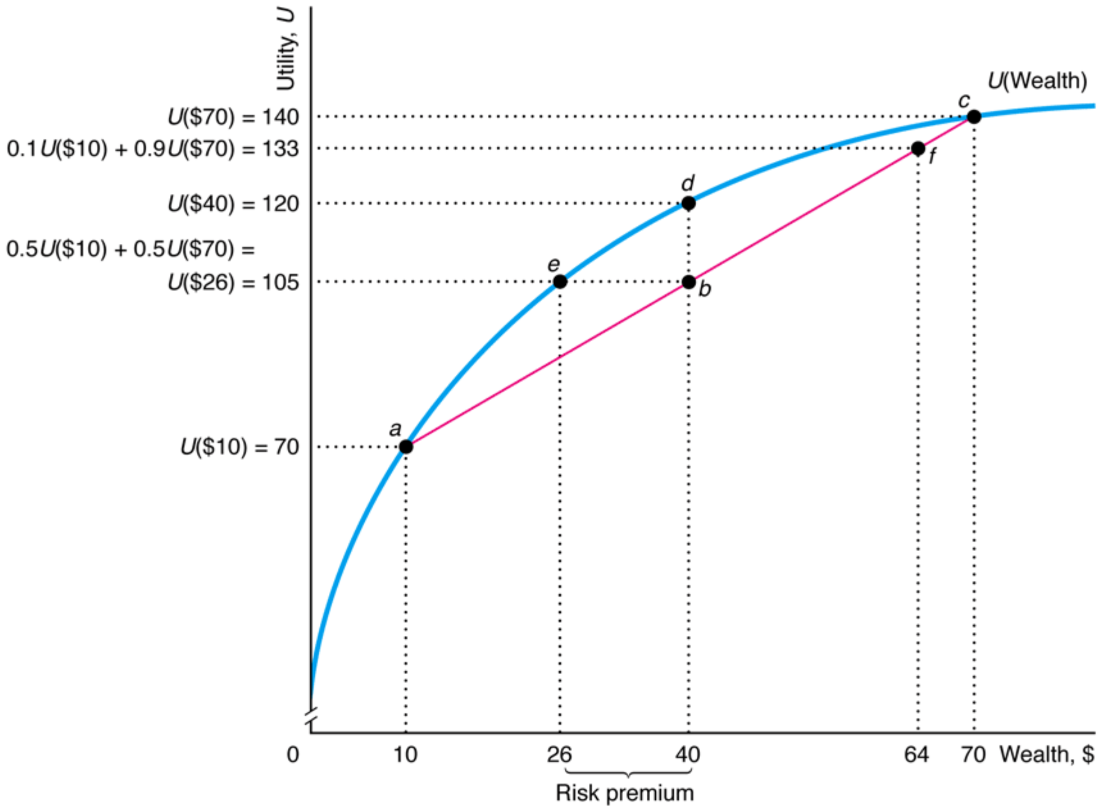
- Diminishing MU (U”<0): “More money is good… but \$1 more when I have \$1,000,000 is not as good as \$1 more when I have \$10!”
  - Dislikes losing more than likes winning.
  - Given the same amount of money, the loss in utility from losing it is greater than the increase in utility from winning it.

2 Attitudes Toward Risk

- Example: Risk-averse Irma and wealth
  - Irma has initial wealth of $40
  - Option 1: keep the $40 and do nothing → $U(\$40) = 120$
  - Option 2: buy a vase that she thinks is a genuine Ming vase with probability of 50%
    - If she is correct, wealth = $70 → $U(\$70) = 140$
    - If she is wrong, wealth = $10 → $U(\$10) = 70$
    - Expected value of wealth remains $\$40 = (½ \cdot \$10) + (½ \cdot \$70)$
    - Expected value of utility is $105 = (½ \cdot 70) + (½ \cdot 140)$
- Although both options have the same expected value of wealth, the option with risk has lower expected utility.

2 Risk Aversion

- She is risk-averse and would pay a ***risk premium*** (14) to avoid risk.
  - Risk premium is the largest amount that a risk-averse person is willing to pay to eliminate the risk and get the expected value with certainty.
- 26 is the **certainty equivalence** (expected value – risk premium) of Option 2: U(CE of Option 2) =EU(Option 2)

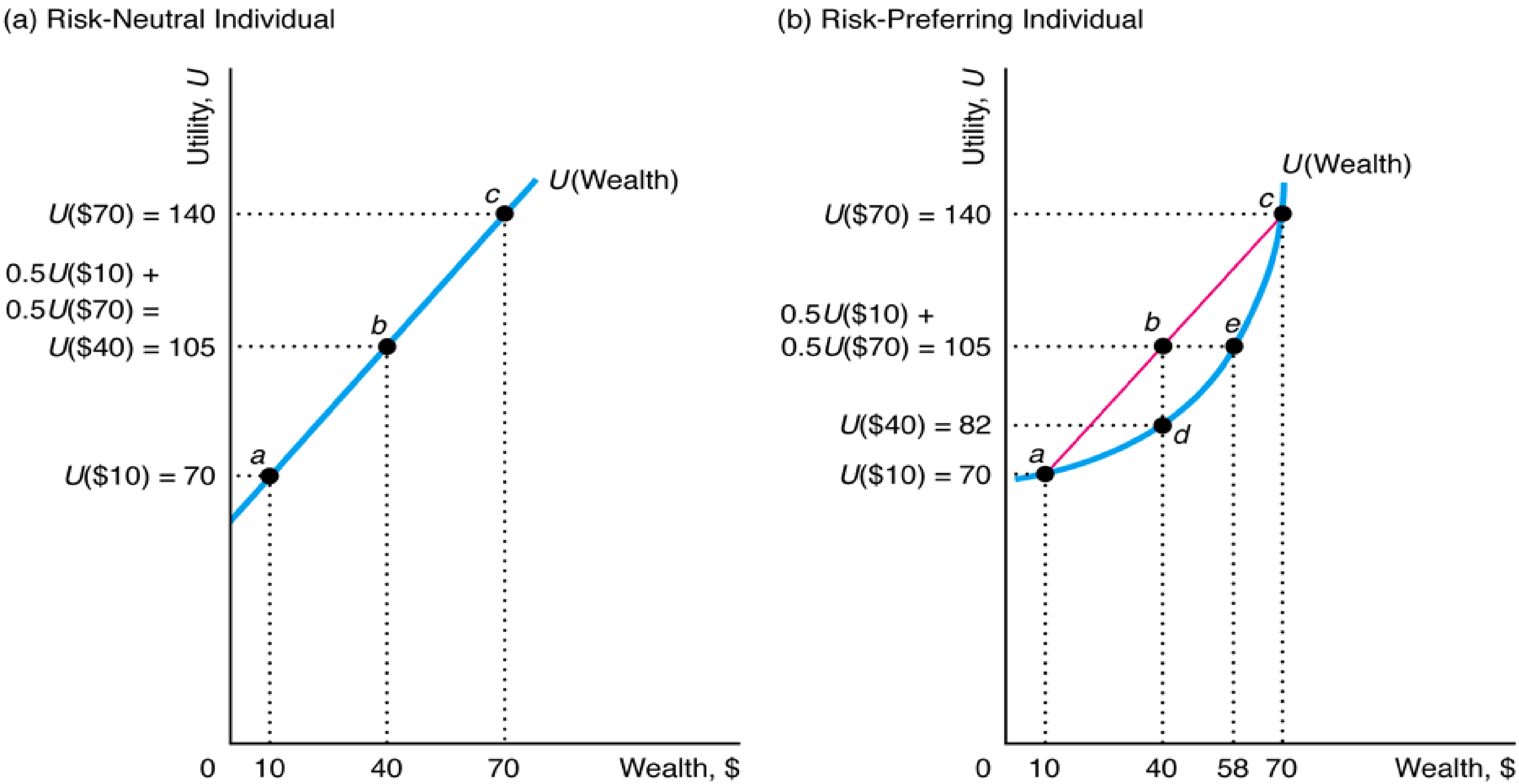


2 Risk Aversion

- Risk premium is the largest amount that a risk-averse person is willing to pay to avoid the risk and get the expected value for certain.
- Certainty equivalence (CE) of a risky option is defined as:
  - U(CE of the option) = EU (the option),
  - meaning that the utility of receiving CE for certain is equal to the expected utility of the risky option.
  - CE=EV- Risk Premium.

2 Risk Neutrality and Risk Preference

- Risk-neutral utility function is a straight line.
- Risk-preferring utility is convex to the horizontal axis.



2 Degree of Risk Aversion

- The degree of risk aversion is judged by the shape of the utility function over wealth, $U(W)$.

- One common measure is the Arrow-Pratt measure of risk aversion:

$$\rho(W) = -\frac{d^2U(W)/dW^2}{dU(W)/dW}$$

  - This measure is positive for risk-averse individuals, zero for risk-neutral individuals, and negative for those who prefer risk.
  - There is a typo in Equation (16.4) of the textbook.

3 Reducing Risk

- There are four primary ways for individuals to avoid risk:

1. **Just say no**
   - Abstaining from risky activities is the simplest way to avoid risk.
2. **Obtain information**
   - Armed with information, people may avoid making a risky choice or take actions to reduce probability of a disaster.
3. **Diversify**
   - “Don’t put all your eggs in one basket.”
4. **Insure**
   - Insurance is like paying a risk premium to avoid risk.

3 Avoiding Risk Via Diversification

- Example: Two firms. Each firm has half chance of being worth \$40, and half chance of being worth \$10. The values of the firms are independent.
- Option 1: Buy 2 shares from one firm

EV=$0.5 \times 80 + 0.5 \times 20 = 50$

Variance: $0.5(80-50)^2+0.5(20-50)^2=900$

- Option 2: Buy 1 share from each firm
  - ¼ chance of both worth \$40, ¼ chance of both worth \$10, ½ chance of one worth \$40 with the other worth \$10

EV=$0.25 \times 80 + 0.5 \times 50 + 0.25 \times 20 = 50$

Variance: $0.25(80-50)^2+0.5(50-50)^2+0.25(20-50)^2=450$, which is lower.

3 Avoiding Risk Via Diversification

- Diversification can reduce risk if two events are independent (uncorrelated).

- Diversification can eliminate risk if two events are *perfectly negatively correlated*.
  - If one event occurs, then the other won't occur.

- Diversification does not reduce risk if two events are *perfectly positively correlated*.
  - If one event occurs, then the other will occur, too.

- Example: investors reduce risk by buying shares in a mutual fund, which is comprised of shares of many companies.

3 Avoiding Risk Via Insurance

- A risk-averse individual will ***fully insure*** by buying enough insurance to eliminate risk if the insurance company offers ***fair insurance***, which eliminates risk but does not change expected income of the individual.
  - In this scenario, the expected value of the insurance is zero; the policyholder's expected value with and without the insurance is the same.

- Insurance companies never offer fair insurance, because they would not stay in business, so most people do not fully insure.

4 Behavioral Economics on Risk and Uncertainty

- Many individuals make choices under uncertainty that are inconsistent with expected utility theory.
  1. Difficulty assessing probabilities => Difficulty assessing expected utility
     - Gambler's fallacy
     - Overconfidence
  2. Behavior varies with circumstances => Factors other than expected utility affect decision
     - Ambiguity aversion
     - Framing effect
     - Certainty effect (Allais paradox)
  3. Prospect theory

4 Difficulty of Assessing Probabilities

- People often have mistaken beliefs about the probability that an event will occur.

- The ***gambler's fallacy*** arises from the false belief that past events affect current, independent outcomes.
  - Example: flipping 'heads' 10 times in a row does not change the probability of getting 'heads' on the next flip from 50%.

- Some people engage in risky gambles because they are overconfident.
  - Surveys of gamblers reveal a big gap between estimated chance of winning a bet and objective probability of winning.

4 Ambiguity

- Two urns, each with 100 red and black balls
- In urn A, there are 50 red balls and 50 black balls.
- In urn B, the composition is unknown.
- Bet 1: a red ball will be drawn from urn A.
- Bet 2: a red ball will be drawn from urn B.
- Which bet do you prefer?
- Most would agree that the subjective probability of drawing a red ball from urn B is 50%.
- But experiments find that more prefer Bet 1.
- **Ambiguity aversion**: People dislike ambiguity.

4 Allais paradox

- **Option A**: receive 4000 with prob. 80%, and 0 with prob. 20%.
- **Option B**: receive 3000 with certainty
- Which would you choose?
- 80% of experimental subjects choose B, the certain outcome.

4 Allais paradox

- **Option C**: receive 4000 with prob. 20%, and 0 with prob. 80%.
- **Option D**: receive 3000 with prob. 25%, and 0 with probability 75%.
- Which would you choose?
- 65% of experimental subjects prefer C.

4 Allais paradox

- The above behavior violates the expected utility theory.

By expected utility theory, choosing B over A means
$U(3000)>0.8U(4000)+0.2U(0)$

$\Rightarrow\ U(3000)-U(0)>0.8[U(4000)-U(0)] \qquad (1)$

Choosing C over D implies
$0.2U(4000)+0.8U(0)>0.25U(3000)+0.75U(0)$

$\Rightarrow\ 0.2[U(4000)-U(0)]+U(0)>0.25[U(3000)-U(0)]+U(0)$

$\Rightarrow\ 0.2[U(4000)-U(0)]>0.25[U(3000)-U(0)]$

$\Rightarrow\ 0.8[U(4000)-U(0)]>U(3000)-U(0) \qquad (2)$

Contradiction between (1) and (2).

4 Allais paradox

- The above behaviour violates independence.
- C=0.25*A+0.75*0; D=0.25*B+0.75*0, where 0 means that receiving 0 with certainty.
  - Option A: 4000 with 80%, and 0 with 20%.
  - Option B: 3000 with certainty
  - Option C: 4000 with 20%, and 0 with 80%.
  - Option D: 3000 with 25%, and 0 with 75%.
- Independence implies that: if B is better than A, then D is better than C.
- The Allais paradox may come from the certainty effect.
- Many people put excessive weight on outcomes they consider to be certain relative to risky outcomes (*certainty effect*).

4 Kahneman and Tversky 1981 (Framing effect)

- An unusual disease is expected to kill 600 people. The gov't is considering two programs, A and B, to combat the disease.
- If A is adopted, 200 people will be saved.
- If B is adopted, with 1/3 prob., 600 people will be saved; with 2/3 prob., no one will be saved.
- Which would you choose?
- In KT's experiment, 72% opted for A over B.

4 Kahneman and Tversky (1981)

- Now the two programs are C and D.
- If C is adopted, 400 people will die.
- If D is adopted, with prob. 1/3, no one will die; with prob. 2/3, 600 people will die.
- Which would you choose?
- In KT’s experiment, 78% opted for D over C.

4 Kahneman and Tversky (1981)

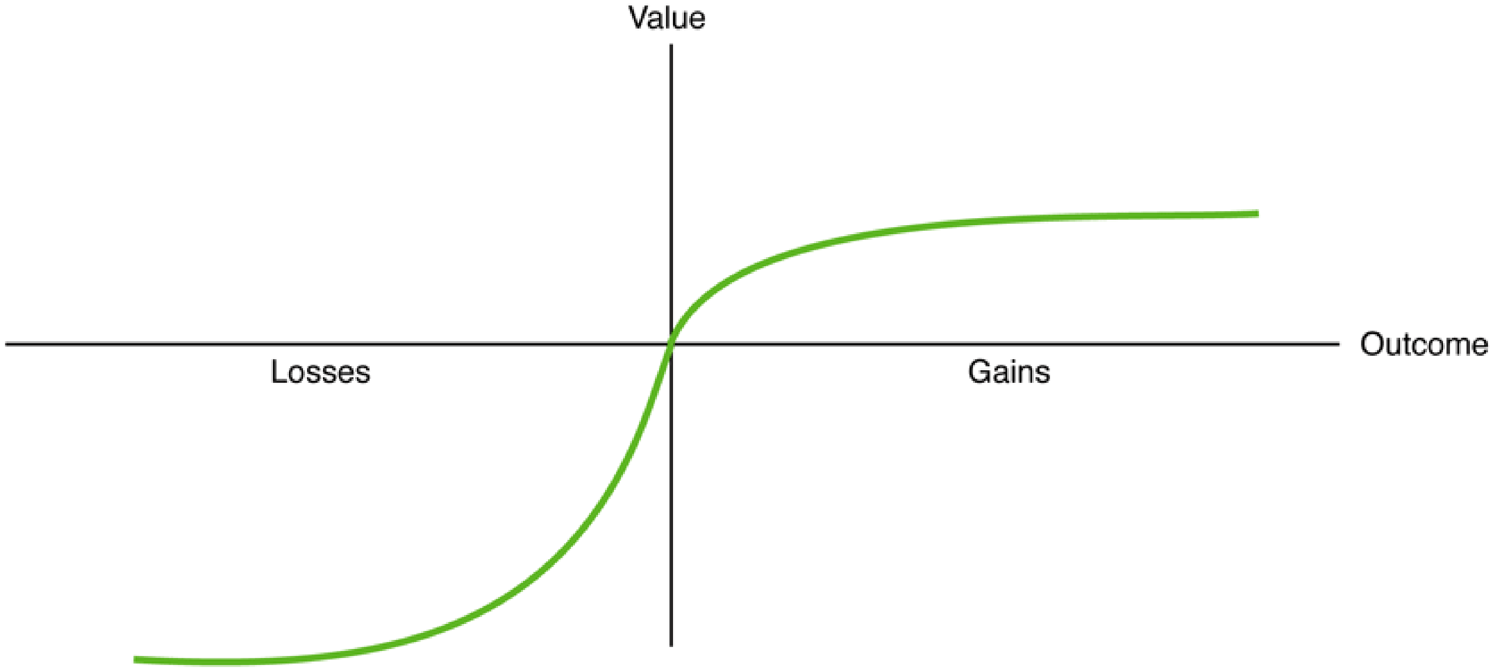
- 72% opted for A over B.
- 78% opted for D over C.
- However, A and C are equivalent and B and D are equivalent.
- Expected utility predicts consistent choices for the two pairs of programs.
- Framing effect: Many people reverse their preferences when a problem is *framed* in a different but equivalent way.
  - People are often risk averse when making choices involving gains, but often risk preferring when making choices involving losses.

4 Prospect Theory

- *Many theories,* called *Non-Expected Utility Theories,* are proposed to explain one or some of the above effects. Out of these, *prospect theory* is most influential.
- *Prospect theory:* People are concerned about gains and losses in wealth (rather than the level of wealth as in expected utility theory)
- The prospect theory value function is S-shaped and has three properties:
  1. **Passes through origin**: gains/losses determined relative to initial situation (reference point)
  2. **Concave to horizontal axis**: less sensitivity to changes in large gains than small ones
  3. **Curve is asymmetric**: people treat gains and losses differently; loss aversion

4 Prospect Theory

- Prospect Theory Value Function



- This can explain the framing effect in the disease example.

Reference:

- Chapter 16:
- Microeconomics: Theory and Applications with Calculus, 3rd Edition. By Jeffrey M. Perloff. 2014 Pearson Education.