

## Lectures 4 \& 5

## Consumer Welfare and Policy Analysis

The welfare of the people is the ultimate law.

Cicero

## Outline

Challenge: Child-Care Subsidies
1 Consumer Welfare
2 Expenditure Function and Consumer Welfare
3 Market Consumer Surplus
4 Effects of Government Policies on Consumer Welfare
5 Deriving Labor Supply Curves
Challenge Solution

## Challenge: Child-Care Subsidies

- Background:
- Government child-care subsidies are common throughout the world.
- Rather than subsidizing the price of child care, the government could provide an unrestricted lump-sum payment that could be spent on child care or on all other goods, such as food and housing.
- Questions:
- For a given government expenditure, does a price subsidy or a lump-sum subsidy provide greater benefit to recipients?
- Which option increases the demand for child-care services more?
- Which one inflicts less cost on other consumers of child care?


## 1 Consumer Welfare

- How much are consumers helped or harmed by shocks that affect the equilibrium price and quantity?
- Shocks may come from new inventions that reduce firm costs, natural disasters, or government-imposed taxes, subsidies, or quotas.
- You might think utility is a natural measure of consumer welfare. Utility is problematic because:
- we rarely know a consumer's utility function
- utility doesn't allow for easy comparisons across consumers
- A better measure of consumer welfare is in terms of dollars.


## 1 Consumer Surplus

- Consumer surplus (CS) is the monetary difference between the maximum amount that a consumer is willing to pay for the quantity purchased and what the good actually costs.
- Step function
(a) David's Consumer Surplus



## 1 Consumer Surplus

- Consumer surplus (CS) is the area under the inverse demand curve and above the market price up to the quantity purchased by the consumer.
- Smooth inverse demand function
(b) Steven's Consumer Surplus



## 1 Effect of a Price Change on Consumer Surplus

- If the price of a good rises (e.g. $£ 0.50$ to $£ 1$ ), purchasers of that good lose consumer surplus (falls by $A+B$ )
- This is the amount of income we would have to give the consumer to offset the harm of an increase in price.



## 1 Effect of a Price Change on Consumer Surplus

## Exercises

1. If the inverse demand function for toasters is $p=60-q$, what is the consumer surplus if the price is 30 ?

Answer
$C S=13,500$

## 2 Expenditure Function and Consumer Welfare

- One measure of the harm to a consumer of a price increase is an increase in the consumer's income needed to maintain the consumer's utility.
- Cannot use an uncompensated demand curve because utility varies along the curve
- Can use compensated demand and the expenditure function because both hold utility constant
- Recall that the minimal expenditure necessary to achieve a specific utility level and given a set of prices is:

$$
E=E\left(p_{1}, p_{2}, \bar{U}\right)
$$

- Welfare change associated with price increase to ${p_{1}}^{*}$ :

$$
\text { welfare change }=E\left(p_{1}, p_{2}, \bar{U}\right)-E\left(p_{1}^{*}, p_{2}, \bar{U}\right)
$$

## 2 Expenditure Function and Consumer Welfare

Lagrangian Method. Another way to solve this constrained maximization problem is to use the Lagrangian method. The Lagrangian expression that corresponds to Problem 3.16 is

$$
\begin{equation*}
\mathscr{L}=U\left(q_{1}, q_{2}\right)+\lambda\left(Y-p_{1} q_{1}-p_{2} q_{2}\right), \tag{3.20}
\end{equation*}
$$

where $\lambda$ (the Greek letter lambda) is the Lagrange multiplier. For values of $q_{1}$ and $q_{2}$ such that the constraint holds, $Y-p_{1} q_{1}-p_{2} q_{2}=0$, so the functions $\mathscr{L}$ and $U$ have the same values. Thus, if we look only at values of $q_{1}$ and $q_{2}$ for which the constraint holds, finding the constrained maximum value of $U$ is the same as finding the critical value of $\mathscr{L}$.

The first-order conditions to find the critical value of $q_{1}, q_{2}$, and $\lambda$ for an interior maximization are

$$
\begin{gather*}
\frac{\partial \mathscr{L}}{\partial q_{1}}=\frac{\partial U}{\partial q_{1}}-\lambda p_{1}=U_{1}-\lambda p_{1}=0 .  \tag{3.21}\\
\frac{\partial \mathscr{L}}{\partial q_{2}}=U_{2}-\lambda p_{2}=0,  \tag{3.22}\\
\frac{\partial \mathscr{L}}{\partial \lambda}=Y-p_{1} q_{1}-p_{2} q_{2}=0 . \tag{3.23}
\end{gather*}
$$

## 2 Expenditure Function and Consumer Welfare

Figure 3.12 Minimizing the Expenditure
The lowest expenditure that Lisa can make that will keep her on indifference curve $I^{2}$ is $E_{2}$. She buys 30 pizzas and 10 burritos.


The solution of this problem is an expression of the minimum expenditure as a function of the prices and the specified utility level:

$$
\begin{equation*}
E=E\left(p_{1}, p_{2}, \bar{U}\right) \tag{3.32}
\end{equation*}
$$

We call this expression the expenditure function: the relationship showing the minimal expenditures necessary to achieve a specific utility level for a given set of prices.

## 2 Expenditure Function and Consumer Welfare

Given that Julia has a Cobb-Douglas utility function $U=q_{1}^{a} q_{2}^{1-a}$, what is her expenditure function?

## Answer

1. Show Julia's Lagrangian function and derive her first-order conditions. Julia's Lagrangian function is $\mathscr{L}=p_{1} q_{1}+p_{2} q_{2}+\lambda\left(\bar{U}-q_{1}^{a} q_{2}^{1-a}\right)$. The first-order conditions for her to minimize her expenditure subject to remaining on a given indifference curve are obtained by differentiating the Lagrangian function with respect to $q_{1}, q_{2}$, and $\lambda$, and setting each derivative equal to zero:

$$
\begin{gather*}
\frac{\partial \mathscr{L}}{\partial q_{1}}=p_{1}-\lambda a q_{1}^{a-1} q_{2}^{1-a}=p_{1}-\lambda a \frac{U}{q_{1}}=0,  \tag{3.33}\\
\frac{\partial \mathscr{L}}{\partial q_{2}}=p_{2}-\lambda(1-a) q_{1}^{a} q_{2}^{-a}=p_{2}-\lambda(1-a) \frac{U}{q_{2}}=0,  \tag{3.34}\\
\frac{\partial \mathscr{L}}{\partial \lambda}=\bar{U}-q_{1}^{a} q_{2}^{1-a}=0 . \tag{3.35}
\end{gather*}
$$

## 2 Expenditure Function and Consumer Welfare

- Which level of utility should be used in this calculation?

$$
\text { welfare change }=E\left(p_{1}, p_{2}, \bar{U}\right)-E\left(p_{1}^{*}, p_{2}, \bar{U}\right)
$$

- Two options:
- Compensating variation is the amount of money we would have to give a consumer after a price increase to keep the consumer on their original indifference curve.
- Equivalent variation is the amount of money we would have to take away from a consumer to harm the consumer as much as the price increase did.


## 2 Compensating Variation and Equivalent Variation

- Indifference curves can be used to determine compensating variation (CV) and equivalent variation (EV).



## 2 Three Measures: CS, CV, and EV

- Relationship between these measures for normal goods:
- $|C V|>|\Delta C S|>|E V|$
- For small changes in price, all three measures are very similar for most goods.



## 2 Three Measures: CS, CV, and EV

## Exercises

1. How much they would have to be paid not to use the Internet?
2. How much they willing to pay to keep using it?

Answer:<br>1. CV<br>2. EV

## 3 Market Consumer Surplus

- Market demand is the (horizontal) sum of individual demand curves; market CS is the sum of each individual's consumer surplus.
- CS losses following a price increase are larger:
- the greater the initial revenue $(p \cdot Q)$ spent on the good
- the less elastic the demand curve at
 equilibrium


## 3 Effect of a 10\% Price Increase on Consumer Surplus

- Revenue and Consumer Surplus in Billions of 2008 Dollars

|  | Revenue | Elasticity of Demand, $\varepsilon$ | Change in Consumer Surplus, $\Delta C S$ |
| :--- | ---: | :---: | :---: |
| Medical | 1,554 | -0.604 | -151 |
| Housing | 1,543 | -0.633 | -149 |
| Food | 669 | -0.245 | -66 |
| Clothing | 338 | -0.405 | -33 |
| Transportation | 301 | -0.461 | -29 |
| Utilities | 308 | -0.448 | -30 |
| Alcohol \& tobacco | 192 | -0.162 | -19 |

## 4 Effects of Government Policies on Consumer Welfare

- Government programs can alter consumers' budget constraints and thereby affect consumer welfare.
- Examples
- Quota: reduces the number of units that a consumer buys
- Subsidy: causes a rotation or parallel shift of the budget constraint
- Welfare programs: may produce kinks in budget constraint


## 4 Effects of Government Policies

- Quotas limit how much of a good consumers can purchase.
- Quota of 12 units generates kink in budget line and removes shaded triangle region from individual's choice set.
- EV of this quota is the income reduction ( $L^{2}$ to $L^{3}$ ) that would move her onto the lower indifference curve, $I^{2}$.



## 4 Effects of Government Policies

- Welfare programs provide either in-kind transfers or a comparable amount of cash to lowincome individuals.
- Example: food stamps
- \$100 in food stamps (inkind) generates kinked budget line.
- \$100 cash transfer increases opportunity set further.


## 4 Effects of Government Policies

- Because food stamps can only be used on food, consumers are potentially worse off if they would find it optimal to consume less food and more other goods than allowed by the program.
- Despite this, food stamps are used rather than comparable cash transfers in order to:
- reduce expenditures on drugs and alcohol
- encourage appropriate expenditure on food from a nutrition standpoint
- maintain program support from taxpayers, who feel more comfortable providing in-kind rather than cash benefits


## 4 Effects of Government Policies

- Subsidies either lower prices or provide lump-sum payments to low-income individuals.
- Example: child care subsidy
- Reducing price of child care rotates budget line out
- Unrestricted lump-sum payment (equal to taxpayers' cost of the subsidy) shifts budget line out in a parallel fashion and increases opportunity set


# 4 Effects of Government Policies on Consumer Welfare 

## TP.HCM sẽ hỗ trợ người bán vé số phải ngưng việc 2 tuần

Các công ty xổ số kiến thiết cũng đã có phương án hồ trợ. Ông Nguyên Thái Bình - giám đốc Công ty TNHH MTV Xổ số kiến thiết Hậu Giang - thông tin trong chiều 30-3 lãnh đạo công ty đã quyết định hỗ trợ ngay 200 tấn gạo cho những người bán vé số dạo thông qua các đại lý cấp 1 .

Ông Trần Minh Khoa - tổng giám đốc Công ty TNHH MTV Xổ số kiến thiết tỉnh Kiên Giang - nói công ty sẽ dành kinh phí nhất định để mua gạo, mì gói, thực phẩm thiết yếu tặng những người bán vé số già cả, neo đơn, hoàn cảnh ngặt nghèo, khuyết tật...

1. Đồng ý việc Công ty TNHH Một thành viên Xố số kiến thiét Trà Vinh trich từ nguồn chi phí kinh doanh đế hồ trỵ cho người bîn lê vê số trên đía bân tînh do ânh hượng của đich bễnh Covid-19, vối mûrc hô try 60.000 đồng/ngày/nguời; thời gian hô trỵ 15 ngày, kể tứr ngày 01/4/2020.
2. Giao Sờ Lao động - Thương bình và Xă hội chù trì, phối hop với Chù tịch UBND cåc huyên, thị xã, thành phố, Công ty TNHH Môt thành viên Xó̀ số kiễn thiêt Trà Vinh khân trưong rà soât, nâm danh sảch nguới bỉn lê vế sồ trễn
 turone./.

4 Effects of Government Policies on Consumer Welfare

## Tăng giá khẩu trang khi có dịch virus corona là đúng hay sai?



## 5 Deriving Labor Supply Curves

- Consumer theory is not only useful for determining consumer demand; it is useful for determining consumers' labor supply decisions.
- Labor - Leisure Choice
- Work ( $H=$ hours) to earn money ( $w=$ wage) and buy goods
- Don't work and consume leisure hours, $N$, and buy goods from unearned income sources, $Y^{*}$
- Utility: $U=U(Y, N)$
- Time constraint: $H=24-N$
- Total income: $Y=w H+Y^{*}$
- Goal in determining labor and leisure choices is to maximize utility subject to constraints.


## 5 Deriving Labor Supply Curves

- Graphical analysis to determine optimal work hours and leisure hours per day:
(a) Indifference Curves and Constraints



## 5 Deriving Labor Supply Curves

- Graphically, when wage falls, it is optimal to work fewer hours and increase leisure:



## 5 Deriving Labor Supply Curves

- Mathematical analysis to determine optimal work hours and leisure hours per day uses calculus to find the tangency point between indifference curve and budget line.
- Maximize utility subject to constraints:

$$
\max _{H} U=U(Y, N)=U(w H, 24-H)
$$

- First-order condition for an interior maximum is:

$$
\frac{\partial U}{\partial Y} \frac{\mathrm{~d} Y}{\mathrm{~d} H}+\frac{\partial U}{\partial N} \frac{\mathrm{~d} N}{\mathrm{~d} H}=U_{\mathrm{Y}} w-U_{N}=0
$$

- Slope of indifference curve $=$ Slope of budget line:

$$
M R S=-\frac{U_{N}}{U_{Y}}=-w=M R T
$$

## 5 Deriving Labor Supply Curves

- The supply curve for hours worked is the mirror image of the demand curve for leisure hours.
(a) Leisure Demand

(b) Labor Supply



## 5 Income and Substitution Effects

- An increase in the wage causes both income and substitution effects.
- Total effect of a wage increase is move from $e_{1}$ to $e_{2}$ (work more).
- Substitution effect is $e_{1}$ to $e^{*}$ (work more).
- Income effect is $e^{*}$ to $e_{2}$ (work less).

- Thus, income effect dominates in this case.


## 5 Leisure is Either an Inferior Good or a Normal Good

- With an increase in income, leisure may increase or decrease

(b) Leisure Inferior



## 5 Shape of the Labor Supply Curve

- Different effects dominate along different portions of the labor supply curve.
- Potentially backward-bending labor supply curve at higher wages

(b) Supply Curve of Labor



## 5 Income Tax Rates and the Labor Supply Curve

- An increase in the income tax rate - a percent of earnings - lowers workers' after-tax wages and may increase or decrease hours worked.
- If labor supply is backward bending, lowering wages through higher income taxes will increase hours worked.
- If labor supply is upward sloping, lowering wages through higher income taxes will decrease hours worked.
- The effect of imposing a marginal tax rate of $\tau$ is to reduce the effect wage from $w$ to $(1-\tau) w$
- This rotates a worker's budget constraint in and downward.


## 5 Income Tax Revenue and Labor Supply

- Income tax revenue is $\tau w H$, which has a nonlinear relationship to the marginal tax rate:



## 5 Income Tax Revenue and Labor Supply

- The government's tax revenue from an income tax is:

$$
T=\tau w H[(1-\tau) w]=\tau w H(\omega)
$$

- Where $H(\omega)$ is the hours of work supplied by an individual given the after tax wage, $\omega=(1-\tau) w$.
- By differentiating the equation above, we can show how income tax revenue changes as the tax rate increases:

$$
\frac{\mathrm{d} T}{\mathrm{~d} \tau}=w H(\omega)-\tau w^{2} \frac{\mathrm{~d} H}{\mathrm{~d} \omega}
$$

- Two effects from a change in the marginal tax rate:

1. Government collects more revenue from higher tax rate.
2. Change in tax rate alters hours worked (and direction cannot be predicted by theory alone).

## Challenge Solution

- Child-care subsidy or lumpsum subsidy?
- Original budget constraint is LO
- If child-care subsidy, budget constraint is $L^{P S}$. Family chooses $e_{2}$ and utility is $I^{2}$.
- If lump-sum subsidy so that $e_{2}$ is affordable, budget constraint is $L^{L S}$. Family chooses $e_{3}$ and utility is $I^{3}$.
- Taxpayer costs for the two programs are the same, but
 family is better off with the lump-sum subsidy.


## 5 Income Tax Revenue and Labor Supply

## Tăng thuế VAT: Cả triệu người ảnh hưởng, Bộ Tài chính 'lùi 1 bước'

[^0]KINH DOANH > TÀI CHÍNH

# Nâng giảm trừ gia cảnh ảnh hưởng thế nào tới thuế thu nhập? 

Quang Thắng đăng lúc 12:20 02/03/2020

## Reference:

- Chapter 5:
- Microeconomics: Theory and Applications with Calculus, 3rd Edition. By Jeffrey M. Perloff. 2014 Pearson Education.


[^0]:    Vhannet Bộ Tài chính đã đề xuất tăng thuế VAT từ $10 \%$ lên $12 \%$ áp dụng từ ngày $1 / 1 / 2019$. Lo ngại tăng thuế sẽ ảnh hưởng mạnh đến toàn bộ đầu tư - sản xuất và lưu thông mà cuối cùng là hàng triệu người dân bị tác động, nhiều bộ, ngành, hiệp hội, địa phươnng có ý kiến không đồng tình. Dẫu vậy, Bộ Tài chính vẫn kiên trì quyết tâm muốn tăng thuế VAT và chỉ "lùi một bước nhỏ".

