# LECTURE 5 <br> FIRMS AND PRODUCTION 

Hard work never killed anybody, but why take a chance?
Charlie McCarthy

## Outline

## Challenge

Labor Productivity During Recessions

1. The Ownership and Management of Firms
2. Production
3. Short Run Production: One Variable and One Fixed Input
4. Long Run Production: Two Variable Inputs
5. Returns to Scale
6. Productivity and Technical Change

Challenge Solution

## Challenge: <br> Labor Productivity During Recessions

## Background

- During a recession, the demand curve for licorice may shift to the left. With reduced demand, mangers of the American Licorice Company have to consider whether to reduce production by laying off some workers.
- The managers must then decide how many workers to layoff.


## Questions

- To make the decision, these managers must consider how much will the output produced per worker rise or fall with each additional layoff?


## 1. The Ownership \& Management of Firms

- A firm is an organization that converts inputs (labor, materials, and capital) into outputs.
- Firm types:
- Private (for-profit) firms: owned by individuals or other non-governmental entities trying to earn a profit (e.g., Toyota, Walmart). Responsible for 75\% of GDP.
- Public firms: owned by governments or government agencies (e.g. Amtrak, public schools). Responsible for $12 \%$ of GDP.
- Not-for-profit firms: owned by organizations that are neither governments nor intended to earn a profit, but rather pursue social or public interest objectives (e.g., Salvation Army, Greenpeace). Responsible for 13\% of GDP.


## The Ownership \& Management of Firms

## Legal forms of organization:

1. Sole proprietorship: firms owned by a single individual who is personally liable for the firm's debts. ( $72 \%$ of firms, but responsible for $4 \%$ of sales)
2. General partnership: businesses jointly owned and controlled by two or more people who are personally liable for the firm's debts. ( $10 \%$ of firms, but responsible for $15 \%$ of sales)
3. Corporation: firms owned by shareholders in proportion to the number of shares or amount of stock they hold. ( $18 \%$ of firms, but responsible for $81 \%$ of sales. Corporation owners have limited liability; they are not personally liable for the firm's debts even if the firm goes into bankruptcy.)

## What Owners Want

- We focus on for-profit firms in the private sector in this course.
- We assume these firms' owners are driven to maximize profit.
- Profit is the difference between revenue (R), what it earns from selling its product, and cost (C), what it pays for labor, materials, and other inputs.

$$
\pi=R-C \text { where } R=p q
$$

- To maximize profits, a firm must produce as efficiently as possible, where efficient production means it cannot produce its current level of output with fewer inputs.


## 2. Production

- The various ways that a firm can transform inputs into the maximum amount of output are summarized in the production function.
- Assuming labor ( L ) and capital ( K ) are the only inputs, the production function is $q=f(L, K)$.
- A firm can more easily adjust its inputs in the long run than in the short run.
- The short run is a period of time so brief that at least one factor of production cannot be varied (the fixed input).
- The long run is a long enough period of time that all inputs can be varied.


## 3. Short Run Production

## One Variable and One Fixed Input

- In the short run (SR), we assume that capital is a fixed input and labor is a variable input.
- SR Production Function: $q=f(L, \bar{K})$
- $q$ is output, but also called total product; the short run production function is also called the total product of labor
- The marginal product of labor is the additional output produced by an additional unit of labor, holding all other factors constant.
- The average product of labor is the ratio of output $A P_{L}=\frac{q}{L}$ to the amount of labor employed.

$$
M P_{L}=\frac{\partial q}{\partial L}=\frac{\partial f(L, K)}{\partial L}
$$

## SR Production with Varlable Labor



## SR Production with Variable Labor

Interpretations of the graphs:

- Total product of labor curve shows output rises with labor until $\mathrm{L}=20$.
- $A P_{L}$ and $M P_{L}$ both first rise and then fall as $L$ increases.
- Initial increases due to specialization of activities; more workers are a good thing
- Eventual declines result when workers begin to get in each other's way as they struggle with having a fixed capital stock
- $M P_{L}$ curve first pulls $A P_{L}$ curve up and then pulls it down, thus, $M P_{L}$ intersects $A P_{L}$ at its maximum.


## Law of Diminishing Marginal Returns

- The law holds that, if a firm keeps increasing an input, holding all other inputs and technology constant, the corresponding increases in output will eventually becomes smaller.
- Occurs at L=10 in previous graph
- Mathematically: $\quad \partial M P_{L} / \partial L=\partial(\partial q / \partial L) / \partial L=$

$$
\partial^{2} q / \partial L^{2}=\partial^{2} f(L, K) / \partial L^{2}<0
$$

- Note that when MPL begins to fall, TP is still increasing.
- LDMR is really an empirical regularity more than a law.
- Application: Malthus and the Green Revolution


## 4. Long Run Production

## Two Variable Inputs

- In the long run (LR), we assume that both labor and capital are variable inputs.
- The freedom to vary both inputs provides firms with many choices of how to produce (labor-intensive vs. capital-intensive methods).
- Consider a Cobb-Douglas production function where A, a, and b are constants:

$$
q=A L^{a} K^{b}
$$

- Hsieh (1995) estimated such a production function for a U.S. electronics firm:

$$
q=L^{0.5} K^{0.5}
$$

## LR Production Isoquants

- A production isoquant graphically summarizes the efficient combinations of inputs (labor and capital) that will produce a specific level of output.



## LR Production Isoquants

- Properties of isoquants:

1. The farther an isoquant is from the origin, the greater the level of output.
2. Isoquants do not cross.
3. Isoquants slope downward.
4. Isoquants must be thin.

- The shape of isoquants (curvature) indicates how readily a firm can substitute between inputs in the production process.


## LR Production Isoquants

Types of isoquants:

1. Perfect substitutes
(e.g., $q=x+y$ )
(a)


## LR Production Isoquants

Types of isoquants:
2. Fixed-proportions
(e.g., $q=\min \{g, b\}$ )
(b)


Cereal per day

## LR Production Isoquants

## Types of isoquants:

3. Convex
(e.g., $q=L^{0.5} K^{0.5}$ )
(c)


## Substituting Inputs

- The slope of an isoquant shows the ability of a firm to replace one input with another (holding output constant).
- Marginal rate of technical substitution (MRTS) is the slope of an isoquant at a single point.

$$
M R T S=\frac{\text { change in capital }}{\text { change in labor }}=\frac{\Delta K}{\Delta L}=\frac{\mathrm{d} K}{\mathrm{~d} L}
$$

- MRTS tells us how many units of $K$ the firm can replace with an extra unit of $L$ (q constant)

$$
\frac{\mathrm{d} \bar{q}}{\mathrm{~d} L}=0=\frac{\partial f}{\partial L}+\frac{\partial f}{\partial K} \frac{\mathrm{~d} K}{\mathrm{~d} L}=M P_{L}+M P_{K} \frac{\mathrm{~d} K}{\mathrm{~d} L}
$$

- MPL = marginal product of labor; MPK = marginal product of capital
- Thus, $M R T S=\frac{\mathrm{d} K}{\mathrm{~d} L}=-\frac{M P_{L}}{M P_{K}}$


## Substituting Inputs

MRTS diminishes along a convex isoquant

- The more $L$ the firm has, the harder it is to replace K with L .



## The Elasticity of Substitution

- Elasticity of substitution measures the ease with which a firm can substitute capital for labor.

$$
\begin{aligned}
\sigma & =\frac{\frac{\mathrm{d}(K / L)}{K / L}}{\frac{\mathrm{~d} M R T S}{M R T S}}=\frac{\mathrm{d}(K / L)}{\mathrm{d} M R T S} \frac{M R T S}{K / L} \\
\sigma & =\frac{\mathrm{d} \ln (K / L)}{\mathrm{d} \ln |M R T S|}
\end{aligned}
$$ derivative:

$=\left(a L^{\rho}+b K^{\rho}\right)^{\frac{d}{p}}$

$$
q=\left(L^{\rho}+K^{\rho}\right)^{\frac{1}{\rho}} \quad \text { MRTS }=-\left(\frac{L}{K}\right)^{\rho-1}
$$

- Constant elasticity:

$$
\sigma=\frac{1}{1-\rho}
$$

## 5. Returns to Scale

- How much does output change if a firm increases all its inputs proportionately?
- Production function exhibits constant returns to scale when a percentage increase in inputs is followed by the same percentage increase in output.
- Doubling inputs, doubles output $\rightarrow \mathrm{f}(2 \mathrm{~L}, 2 \mathrm{~K})=2 \mathrm{f}(\mathrm{L}, \mathrm{K})$
- More generally, a production function is homogeneous of degree $\gamma$ if $f(x L, x K)=x^{\vee} f(L, K)$ where $x$ is a positive constant.


## Returns to Scale

- Production function exhibits increasing returns to scale when a percentage increase in inputs is followed by a larger percentage increase in output.
- $f(2 L, 2 K)>2 f(L, K)$
- Occurs with greater specialization of $L$ and $K$; one large plant more productive than two small plants
- Production function exhibits decreasing returns to scale when a percentage increase in inputs is followed by a smaller percentage increase in output.
- $f(2 L, 2 K)<2 f(L, K)$
- Occurs because of difficulty organizing and coordinating activities as firm size increases.


## Varying Returns <br> to Scale



## Returns to Scale Estimates in U.S. Manufacturing

|  | Labor, $a$ | Capital, $b$ | Scale, $\gamma=a+b$ |
| :--- | :---: | :---: | :---: |
| Decreasing Returns to Scale |  |  |  |
| Tobacco products | 0.18 | 0.33 | 0.51 |
| Food and kindred products | 0.43 | 0.48 | 0.91 |
| Transportation equipment | 0.44 | 0.48 | 0.92 |
| Constant Returns to Scale |  |  |  |
| Apparel and other textile products | 0.70 | 0.31 | 1.01 |
| Furniture and fixtures | 0.62 | 0.40 | 1.02 |
| Electronic and other electric | 0.49 | 0.53 | 1.02 |
| equipment |  |  |  |
| Increasing Returns to Scale | 0.44 | 0.65 | 1.09 |
| Paper and allied products | 0.30 | 0.88 | 1.18 |
| Petroleum and coal products | 0.51 | 0.73 | 1.24 |
| Primary metal |  |  |  |

## 6. Productivity and Technical Change

- Even if all firms are producing efficiently (an assumption we make in this lecture), firms may not be equally productive.
- Relative productivity of a firm is the firm's output as a percentage of the output that the most productive firm in the industry could have produced with the same inputs.
- Relative productivity depends upon:

1. Management skill/organization
2. Technical innovation
3. Union-mandated work rules
4. Work place discrimination
5. Government regulations or other industry restrictions
6. Degree of competition in the market

## 6. Productivity and Technical Change

- An advance in firm knowledge that allows more output to be produced with the same level of inputs is called technical progress.
- Example: Nano by Tata Motors
- Neutral technical change involves more output using the same ratio of inputs.
- Non-neutral technical change involves altering the proportion in which inputs are used to produce more output.
- Organizational change may also alter the production function and increase output.
- Examples: automated production of Gillette razor blades, mass production of Ford automobiles


## Challenge Solution

- Do layoffs at licorice plants result in increase labor productivity? If the graph below, if the plant employs fewer than 15 workers, then APL falls with a layoff. But if the firm employs more than 15 workers, then APLincreases.
- One estimated production function for food plants is for this production function:


$$
\mathrm{q}=\mathrm{AL}^{0.43} \mathrm{~K}^{0.48} \mathrm{APL}
$$

For this production function APL would increase because $\partial \mathrm{APL} / \partial \mathrm{L}=(-0.57) \mathrm{AL}^{-1.57} \mathrm{~K}^{0.48}<0$.


## Microeconomics with Calculus

THIRD EDITION<br>Jeffrey M. Perloff



## REFERENCE

Chapter 6 - Microeconomics: Theory and Applications with Calculus, 3rd Edition. By Jeffrey M. Perloff. 2014 Pearson Education.

