Descriptive Statistics

Outline

- Introduction
- Data types
- Measures of centre
- Measures of variability
- Practical meanings of standard deviation
- Measures of relative standing
- Measures of skewness
- Describing bivariate numerical data

Introduction

- **Descriptive statistics** “summarise and describe the important characteristics of a set of measurements”

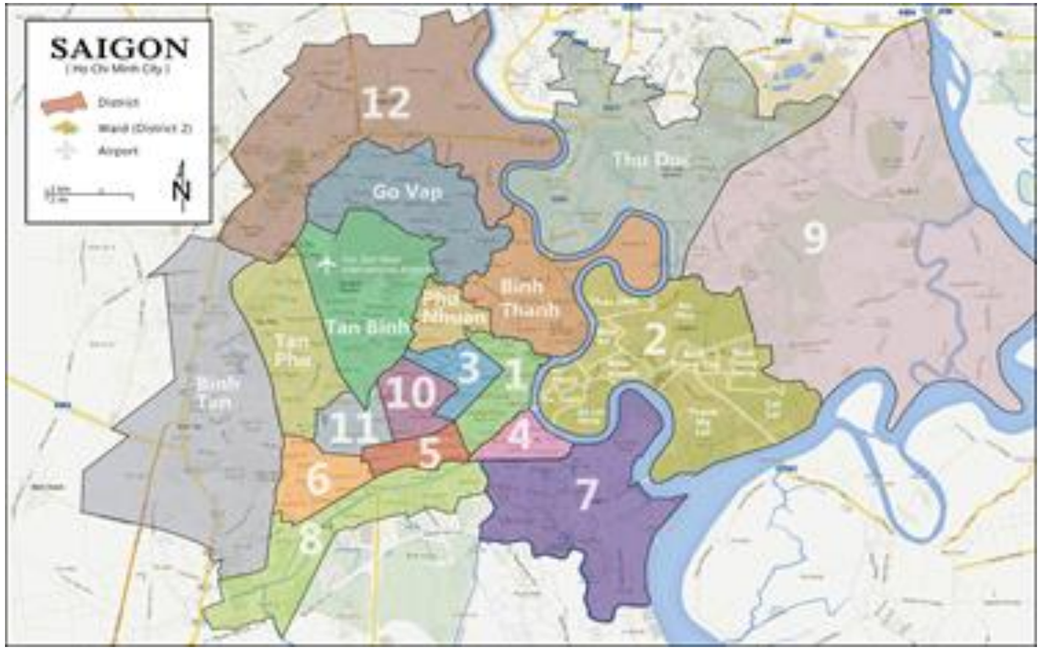
- **Inferential statistics** “make inferences about **population** characteristics from information contained in a **sample** drawn from this population”

Structured data vs unstructured data

Unstructured data – signals, images, text, graphs, sounds

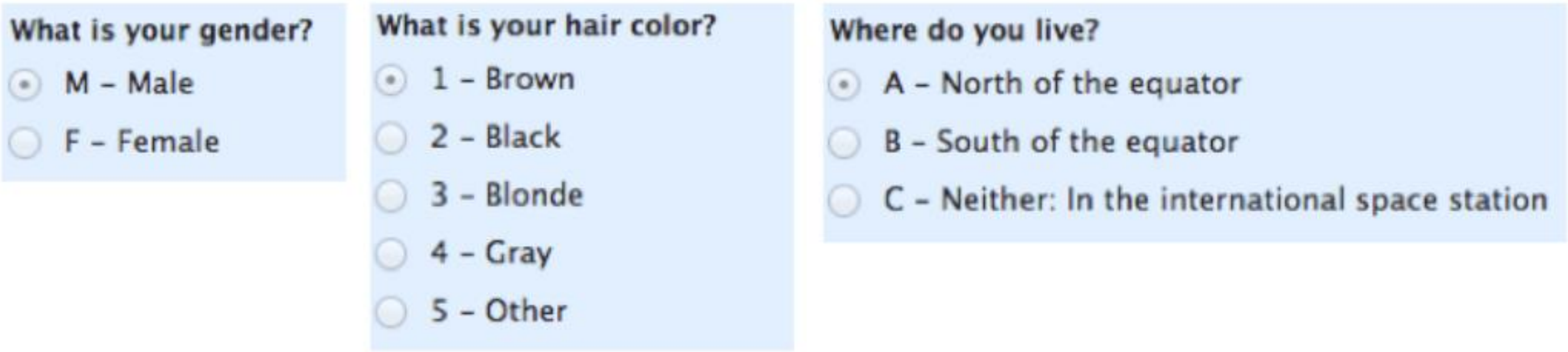
Structured data – cross-sectional, panel, time series

- Data types: nominal, interval, ratio, transaction, latitude/longitude, shapefile



Data types

- **Nominal:** labels, mutually exclusive, no numerical significance, may or may not have orders

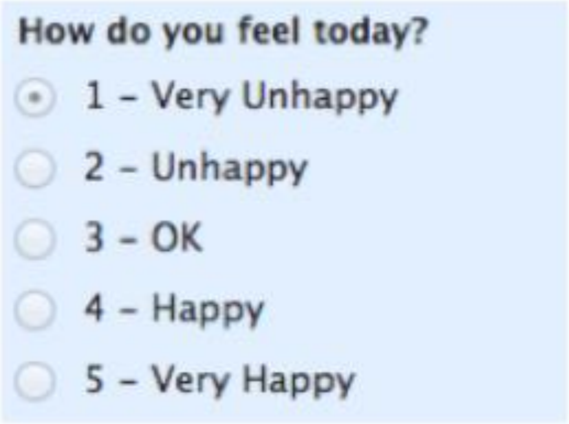


Source: https://www.mymarketresearchmethods.com/types-of-data-nominal-ordinal-interval-ratio/#targetText=Summary,the%20difference%20between%20each%20one.

Data types

- **Ordinal:** in order but the difference between variables not defined, e.g. Likert scales, time of day (morning, noon, evening), energy rating (1 star, 2 stars, 3 stars)

  Example. Likert scales – Very Happy is better (higher) than Happy. The difference between Very Happy and Happy doesn't make sense, and does not equal the difference between OK and Unhappy.

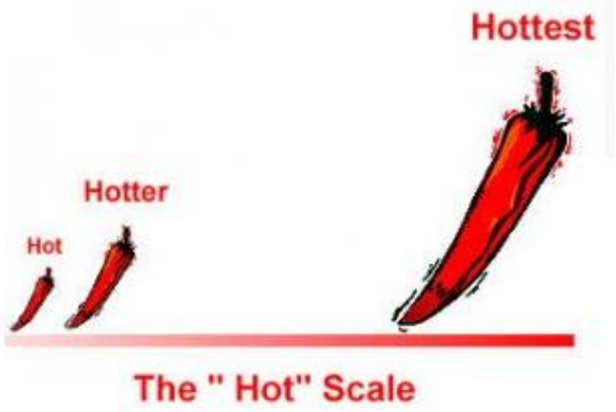


How do you feel today?

1 – Very Unhappy
2 – Unhappy
3 – OK
4 – Happy
5 – Very Happy

How satisfied are you with our service?

1 – Very Unsatisfied
2 – Somewhat Unsatisfied
3 – Neutral
4 – Somewhat Satisfied
5 – Very Satisfied



Source: https://www.mymarketresearchmethods.com/types-of-data-nominal-ordinal-interval-ratio/#targetText=Summary,the%20difference%20between%20each%20one.
https://www.statisticshowto.datasciencecentral.com/nominal-ordinal-interval-ratio/.

Data types

- **Interval:** in order, difference between variables defined, but don’t have a “true zero” and thus cannot be divided or multiplied, e.g. temperature, time on a clock, IQ score.

  Example. Temperature - water from 20^{o}C needs an increase of 80^{o}C to 100^{o}C to boil, but 0^{o}C does not mean water has **no** temperature. Also, 80^{o} is not 4 times of 20^{o} because 0^{o} is not a starting/reference point.

- **Ratio**: like interval but with a “true zero”, e.g. income, years of education, weight.

Source: https://www.mymarketresearchmethods.com/types-of-data-nominal-ordinal-interval-ratio/#targetText=Summary,the%20difference%20between%20each%20one.
https://www.statisticshowto.datasciencecentral.com/nominal-ordinal-interval-ratio/.

Data type	Mathematical operations	Measures of central tendency	Measures of variability
Nominal	• Equality (=, ≠)	• Mode	• None
Ordinal	• Equality (=, ≠) • Comparison (>, <)	• Mode • Median	• Range • Interquartile range
Interval	• Equality (=, ≠) • Comparison (>, <) • Addition, subtraction (+, −)	• Mode • Median • Arithmetic mean	• Range • Interquartile range • Standard deviation • Variance
Ratio	• Equality (=, ≠) • Comparison (>, <) • Addition, subtraction (+, −) • Multiplication, division (×, ÷)	• Mode • Median • Arithmetic mean • *Geometric mean	• Range • Interquartile range • Standard deviation • Variance • **Relative standard deviation

Source: https://www.scribbr.com/statistics/levels-of-measurement/

Data types – Practice Example

What is the type of these variables?

Features	Value set	Unit
Electric vehicle properties		
Vehicle type	Large sedan, Minivan, Small sedan, Large SUV, Small SUV, Small hatchback	
Range	120, 180, 240, 300, 360, 420, 480, 540	km
Recharge time	0.5, 1.5, 2.5, 3.5, 4.5, 5.5, 6.5, 7.5	hours
Set up cost	1000, 1750, 2500, 3250	Dollars
Cost per km	3, 6, 9, 12	Cents
EV price	25000, 35000, 45000, 55000, 70000, 85000, 100000, 120000, 140000, 160000	Dollars
Governmental supports		
Charging station availability	5, 10, 15, 20	km
Bus lane access	Access to bus lane, No access to bus lane	
Rebates upfront costs	0, 3000, 6500, 10000	Dollars
Rebates parking fees	0, 100, 250, 400	Dollars
Energy bill discount	0, 25, 50, 75	Percent
Stamp duty discount	0, 5, 15, 25	Percent
Market penetration stage (in NSW)		
Percentage EV sold	1, 30, 60, 90	Percent

Features	Value set	Unit
Gender	Male, Female	
Annual gross household income	Continuous value	Dollars
Number of cars in household	0, 1, 2, more than 2	cars
Number of other driver licences in household	Continuous value	
Currently hold a driver licence	Yes, No	
Household type	Couple family with no children, Couple family with children, One parent family, Single person household, Group household, Other family	
Work status	Employed full time, Employed part time, Household duties, Retired, Student, Unemployed	

Measures of Centre

- **Sample mean ($\bar{x}$):** $\bar{x} = \frac{\sum x_i}{n}$
  - What is the sample mean of [2, 9, 11, 5, 6, 27]?
  - What is the sample mean of [2, 9, 110, 5, 6, 27]?

- **Population mean (μ)**: usually unknown, estimated by $\bar{x}$

- **Median (m)**:
  - The value of x that falls in the middle position of an ordered sample
  - Median value $\mathrm{m} = \boldsymbol{x}_{\mathbf{0.5(n+1)}}$

    $2 \quad 5 \quad 6 \quad 9 \quad 11$

    $2 \quad 5 \quad \boxed{6 \quad 9} \quad 11 \quad 27$

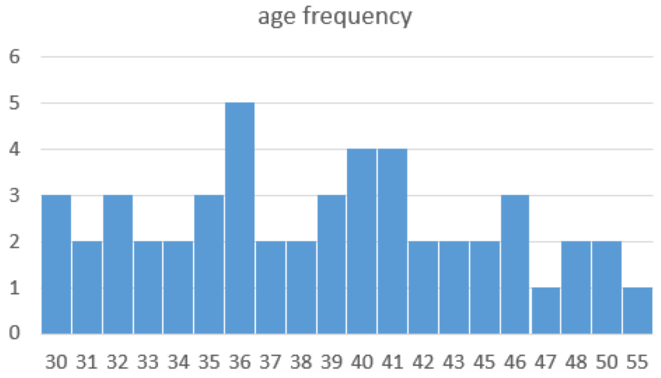
  - What is the median of [2, 9, 110, 5, 6, 27]?

  -> Less sensitive to outliers

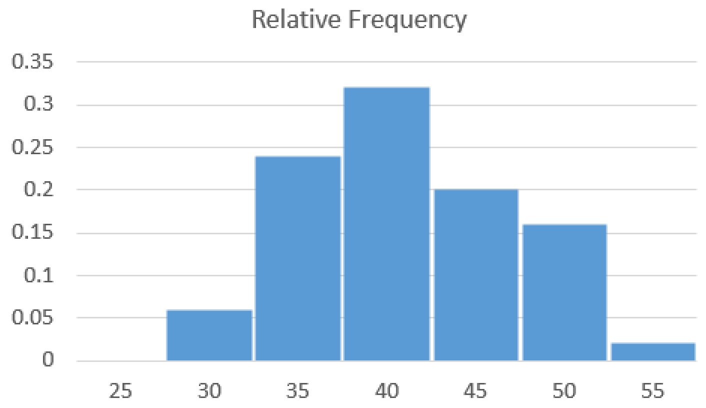
Measures of Centre

- **Mode:** "the category that occurs most frequently, or the most frequently occurring value of x"
- Relative frequency plot
  - Example: The ages (in months) at which 50 kids were first enrolled in a preschool

38	40	30	35	39	40	48	36	31	36
47	35	34	43	41	36	41	43	48	40
32	34	41	30	46	35	40	30	46	37
55	39	33	32	32	45	42	41	36	50
42	50	37	39	33	45	38	46	36	31



Bin	*Frequency*	*Relative Frequency*
25	0	0
30	3	0.06
35	12	0.24
40	16	0.32
45	10	0.2
50	8	0.16
55	1	0.02
Total	50	1



- Mode is generally used for large data sets, whereas mean and median can be used for any.

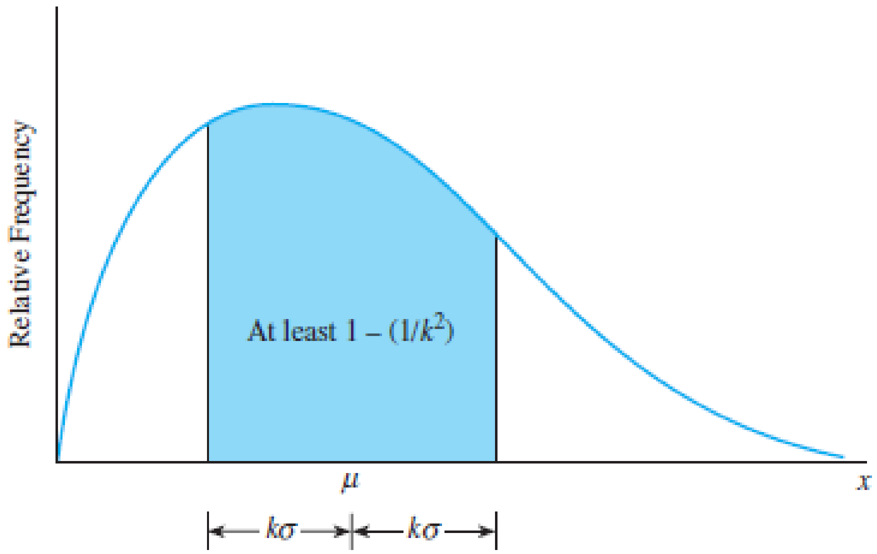
Measures of Variability

- **Range ($\boldsymbol{R}$):** "the difference between the largest and smallest measurements"
- **Deviation:** difference between the sample mean and a measurement x_i, $x_i - \bar{x}$
- **Variance of a sample:** $s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$
- **Variance of a population:** $\sigma^2 = \frac{\sum(x_i - \bar{x})^2}{N}$
- **Standard deviation:** square root of the variance

Practical meanings of standard deviation

Tchebysheff's Theorem. For **any** dataset

- **At least** none of the measurements lie in the interval $\mu \pm \sigma$
- **At least** 3/4 (75%) of the measurements lie in the interval $\mu \pm 2\sigma$
- **At least** 8/9 (88.9%) of the measurements lie in the interval $\mu \pm 3\sigma$



Practical meanings of standard deviation

Example. The ages (in months) at which 50 kids were first enrolled in a preschool

38	40	30	35	39	40	48	36	31	36
47	35	34	43	41	36	41	43	48	40
32	34	41	30	46	35	40	30	46	37
55	39	33	32	32	45	42	41	36	50
42	50	37	39	33	45	38	46	36	31

Mean = 39.08 months, std = 5.99 months

- Tchebysheff's theorem:

  At least ¾ of the kids (37.5 kids) are from 27.11 months to 51.05 months ($\mu \pm 2\sigma$)
- Fact: 49 kids are from 33.09 months to 45.07 months.

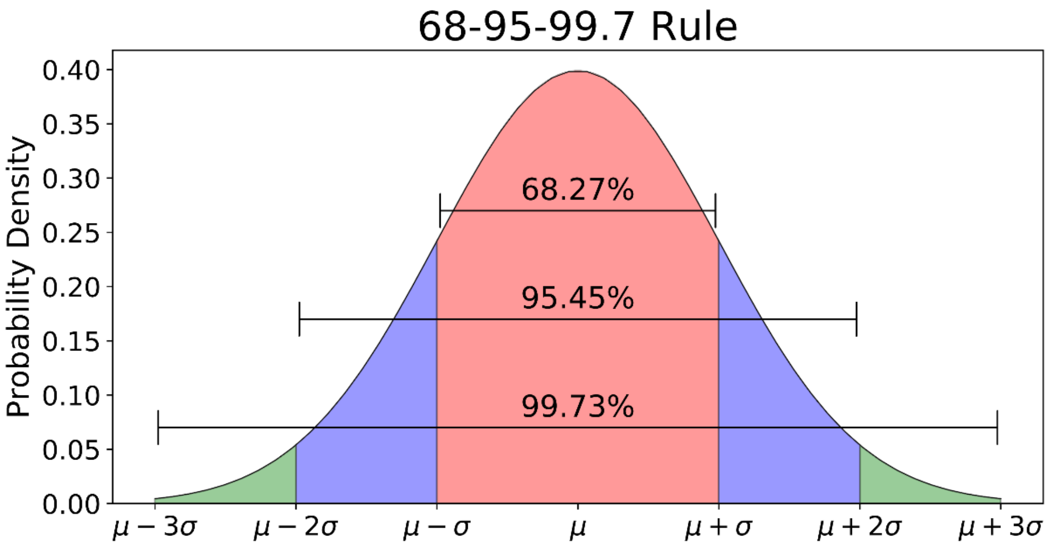
- Tchebysheff's theorem:

  At least 8/9 of the kids (44.4 kids) are from 21.12 months to 57.04 months ($\mu \pm 3\sigma$)
- Fact: 50 kids are from 33.09 months to 45.07 months.

Practical meanings of standard deviation

The Empirical Rule. For an **approximately normal distribution** of measurements

- 68% of the measurements lie in the interval $\mu \pm \sigma$
- 95% of the measurements lie in the interval $\mu \pm 2\sigma$
- 99.7% of the measurements lie in the interval $\mu \pm 3\sigma$

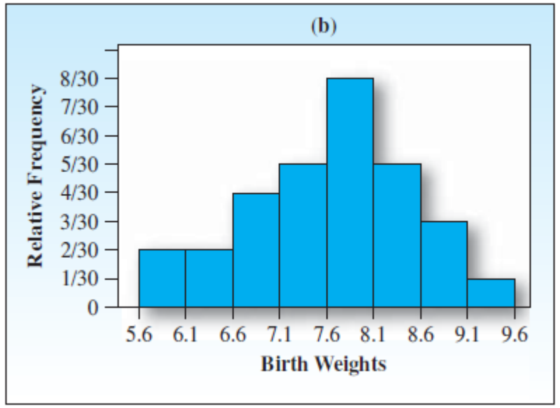


[Source: https://towardsdatascience.com/understanding-the-68-95-99-7-rule-for-a-normal-distribution-b7b7cbf760c2](https://towardsdatascience.com/understanding-the-68-95-99-7-rule-for-a-normal-distribution-b7b7cbf760c2)

Practical meanings of standard deviation

Example. Birth weights (in pounds) of 30 full-term new born babies

7.2	7.8	6.8	6.2	8.2
8.0	8.2	5.6	8.6	7.1
8.2	7.7	7.5	7.2	7.7
5.8	6.8	6.8	8.5	7.5
6.1	7.9	9.4	9.0	7.8
8.5	9.0	7.7	6.7	7.7



Mean = 7.57 lbs, std = 0.95 lbs

The Empirical Rule: At least 68% of the babies (20.4 babies) are from 6.63 lbs to 8.52 lbs ($\mu \pm \sigma$)

Facts: 22 babies have weights between 6.63 lbs and 8.52 lbs.

The Empirical Rule: At least 95% of the babies (28.5 babies) are from 5.68 lbs to 9.47 lbs ($\mu \pm 2\sigma$)

Facts: 29 babies have weights between 5.68 lbs and 9.47 lbs.

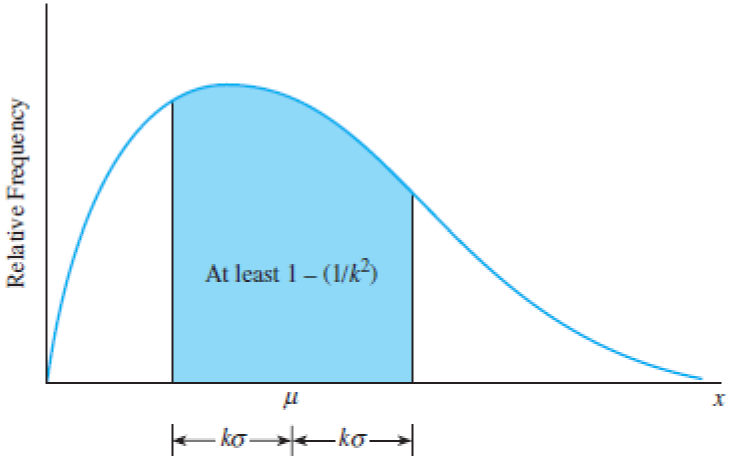
Measures of Relative Standing

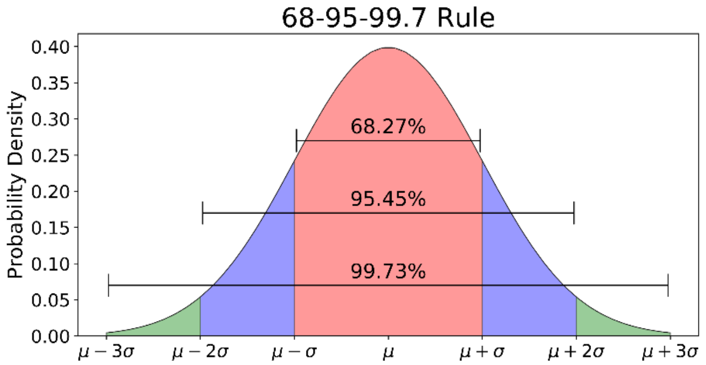
Sample z-score

- “distance between an observation and the mean measured in units of standard deviation”

$$zscore = \frac{x - \bar{x}}{s}$$

- A valuable tool in determining outliers. If z-score < -3 or z-score > 3 => outliers.

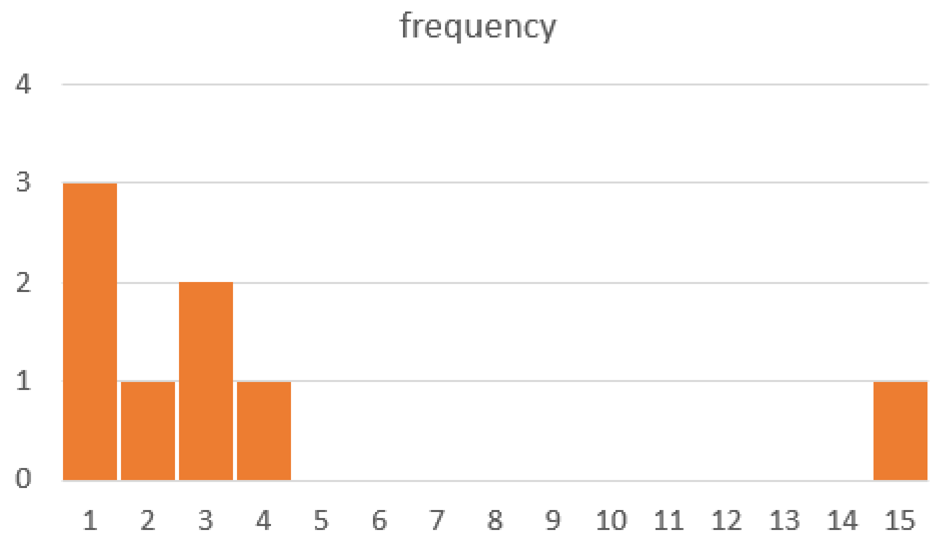




Measures of Relative Standing

Example. Calculate z-score of each observation for potential outliers in the list of measurements of [1, 1, 0, 15, 2, 3, 4, 0, 1, 3].

- Mean = 3, std = 4.42
- Z-score of x=15 is $\frac{15-3}{4.42} = 2.72$
- 15 may be considered as an outlier

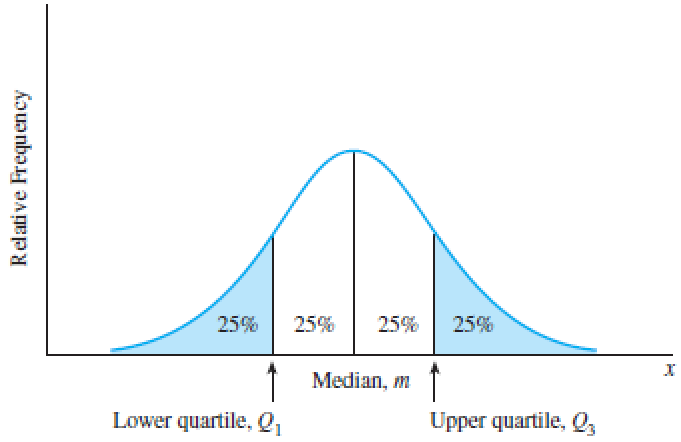


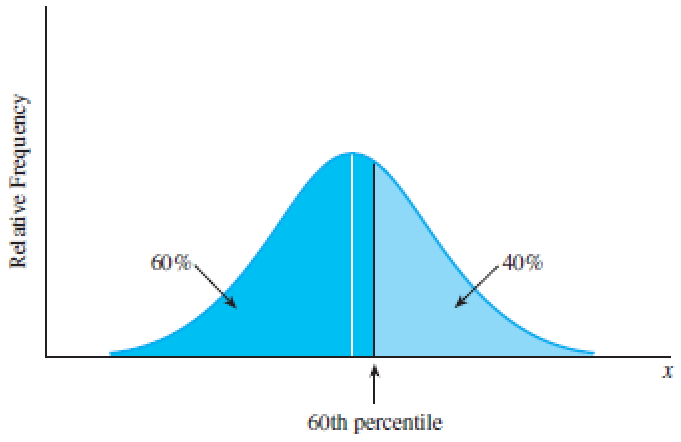
Measures of Relative Standing

***p*th percentile** "is the value of x that is greater than p% of the (ordered) measurements and is less than the remaining (100-p)%"

Percentile of value x = (number of values less than x)/(number of values)*100

Lower quartile, **upper quartile** and **interquartile range**





Position of Q1 value is calculated by 0.25*(n+1)

Position of Q3 value is calculated by 0.75*(n+1)

Measures of Relative Standing

Example. Consider the set of measurements [16, 25, 4, 18, 11, 13, 20, 8, 11, 9]

- Sort the measurements [4, 8, 9, 11, 11, 13, 16, 18, 20, 25]

- Value 18 is at 70^{th} percentile

- Position of the 25^{th} percentile is 0.25*(10+1) = 2.75.

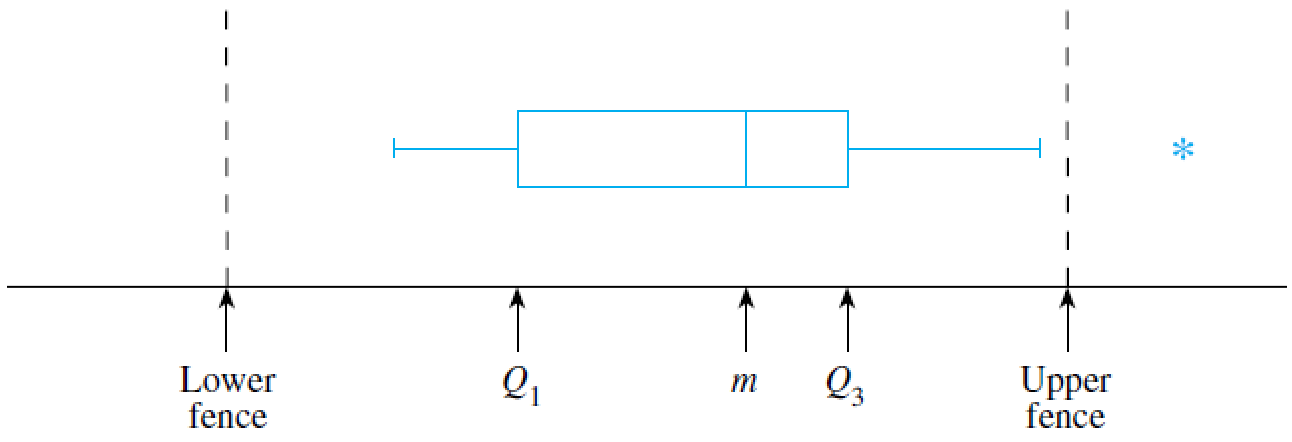
Q1 value is therefore 8 + .75*(9-8) = 8.75

- Position of the 75^{th} percentile is 0.75*(10+1) = 8.25.

Q3 value is therefore 18 + .25(20-18) = 18.5

The 5-number summary and Box Plots

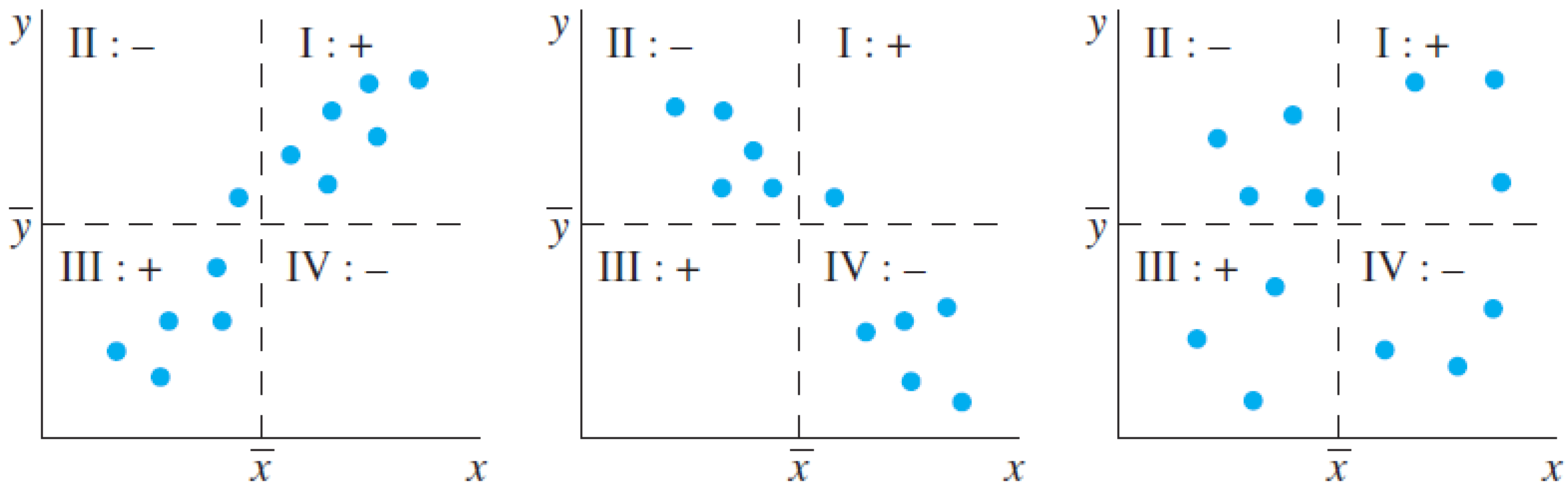
- Five-number summary: Min, Q1, Median, Q3, Max
- A graphical tool “expressly designed” for isolating outliers from a sample.



- Lower fence = Q1 – 1.5(IQR)
- Upper fence = Q3 + 1.5(IQR)

Describing Bivariate Numerical Data

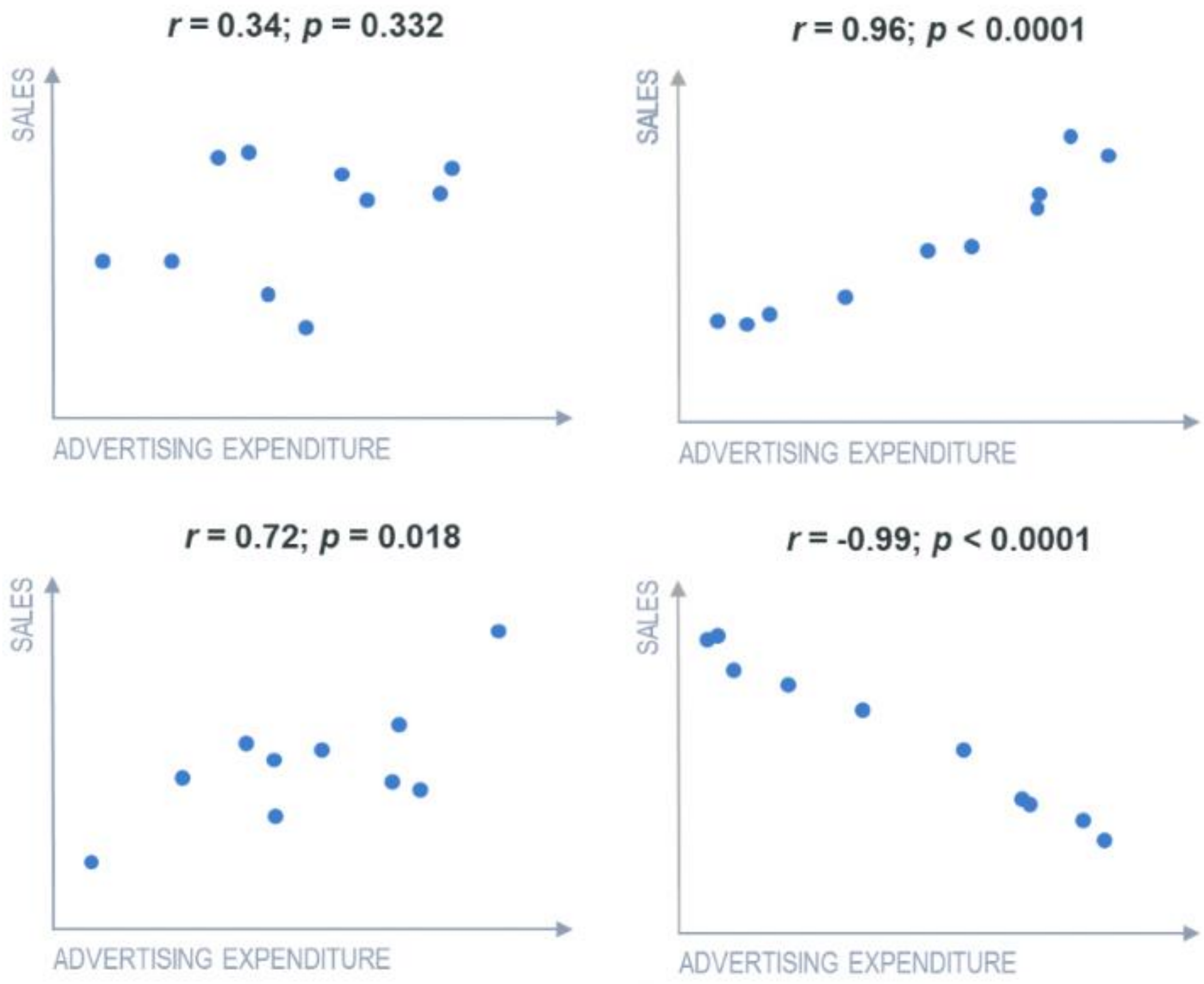
- Covariance between x and y in a bivariate sample, $s_{xy} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{n-1}$
- Correlation coefficient, $r = \frac{s_{xy}}{s_x s_y}$

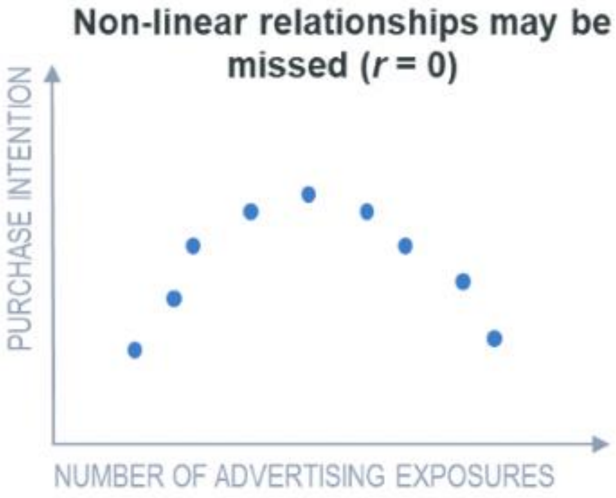


(a) Positive pattern (b) Negative pattern (c) No pattern

Describing Bivariate Numerical Data

- Correlation coefficient $-1 \leq r \leq 1$, indicating the strength of the correlation
- $r = 1$: perfect positive correlation
- $r = -1$: perfect negative correlation
- $r = 0$: no correlation between x and y (?)





[Source: https://www.displayr.com/what-is-correlation/](https://www.displayr.com/what-is-correlation/)

Review

- Descriptive statistics and inferential statistics
- Sample vs Population
- Data types: nominal, ordinal, interval, ratio
- Measure of Centre: Mean, Median, Mode
- Measure of Variability: Range, Deviation, Variance, Standard Deviation
- Tchebysheff's Theorem, the Empirical Rule, and outlier detection
- Measures of relative standing: p^{th} percentile, quartiles, interquartile range
- Box plots
- Describing bivariate data: covariance and correlation coefficient