Large-Sample Tests of Hypotheses (Part 1)

Outline

- A statistical test of hypothesis
- Large sample test about a population mean

A statistical test of hypothesis

Five components of a statistical test

(1) The **null hypothesis, H_0**

(2) The **alternative hypothesis, H_a**

(3) The **test statistic** and its **p-value**

(4) The **rejection region**

(5) The **conclusion**

A statistical test of hypothesis

- (1) The **null hypothesis, H_0**

  The hypothesis contradicting H_a, e.g. H_0: $\mu = \$456$
- (2) The **alternative hypothesis, H_a**

  The hypothesis that we wish to support, for example

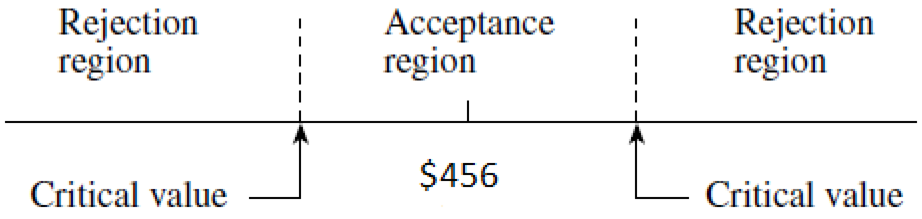
  Example 1: H_a: $\mu \neq \$456$ (2-tailed test of hypothesis)

  Example 2: H_a: $\mu < \$456$ (1-tailed test of hypothesis)

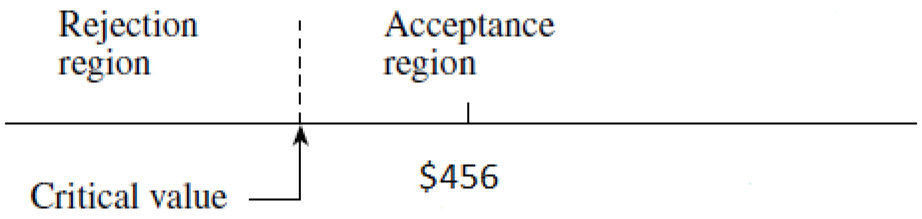
  Example 3: H_a: $\mu > \$456$ (1-tailed test of hypothesis)

A statistical test of hypothesis

- (3) **Test statistic** is a single value calculated from the sample data and **p-value** is a probability of observing an example as large (or as small) as the test statistic.
- (4) The set of possible values of test statistic can be divided into 2 regions
  - **Rejection region** – includes values that support the alternative hypothesis H_a and rejects the null hypothesis H_0
  - **Acceptance region** – includes values that support the null hypothesis H_0



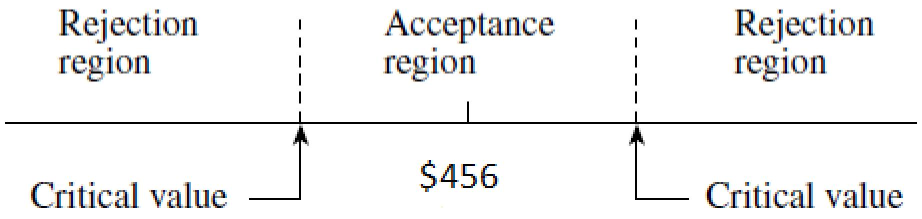
H_a: $\mu \neq \$456$



H_a: $\mu < \$456$

A statistical test of hypothesis

- (5) **Conclusions** – we *always* begin with assuming that the null hypothesis is true, then use sample data as evidence to decide one of the 2 conclusions
  - Reject H_0 and conclude H_a is true
  - Accept H_0 as true or the test is inconclusive



- The critical values are decided based on the **significance level α**, which represents the probability of rejecting H_0 when it is true.
- **Type I error** – the error of rejecting the null hypothesis when it is true.

A large-sample test about a population mean

Example – The average monthly income of people in HCMC is \$456. A random sample of n=51 IT professionals in HCMC showed that average income $\bar{x} = \$500$, with standard deviation $s = \$155$. Do IT professionals have higher monthly income than the city average? Test the hypothesis with significance level $\alpha = .05$ (or 5%).

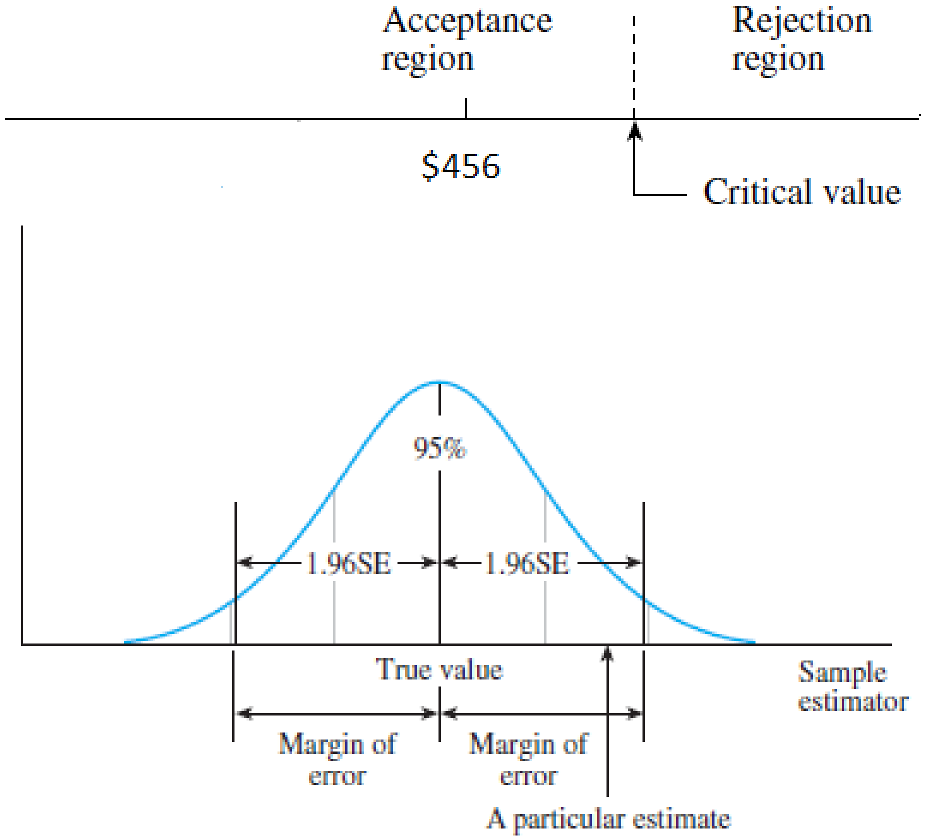
- (1) The **null hypothesis, H_0**: $\mu = \$456$
- (2) The **alternative hypothesis, H_a**: $\mu > \$456$

A large-sample test about a population mean

- Because n is fairly large, the sample mean $\bar{x} = \$500$ is the best estimate of the true average income μ of IT professionals in HCMC (the Central Limit Theorem).

- How large $\bar{x}$ needs to be compared to $\mu_0 = \$456$ for us to reject the null hypothesis?

- Because the sampling distribution of $\bar{x}$ follows a normal distribution, the mean of which is μ, if μ_0 is *many standard errors* (SEs) away from μ we can fairly sure that the probability to see μ_0 is very low, i.e. μ_0 does not equal μ.



A large-sample test about a population mean

- But how many SEs are enough? We need to rely on the significance level α.
- Standard error of $\bar{x}$, $SE = \frac{s}{\sqrt{n}} = \frac{155}{\sqrt{51}} = \$21.9$
- (3) **Test statistic**: The number of SEs $\mu_0 = \$456$ is away from $\bar{x}$ is calculated by

$$\mathrm{z} = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{500-456}{21.9} = 2.03$$

In other words, $\bar{x} = \mu_0 + 2.03 * SE$.

- (4) Rejection region: For significance level $\alpha = .05$, the corresponding z-score is 1.64. Any observed z-value larger than this will be in the rejection region.
- (5) Conclusions: Because the test statistic z = 2.03 is larger than the critical value of 1.64, we reject the null hypothesis, and conclude that the *average monthly income of IT professionals is higher than the city average*.
- The probability of this conclusion being wrong is $\alpha = 5\%$.

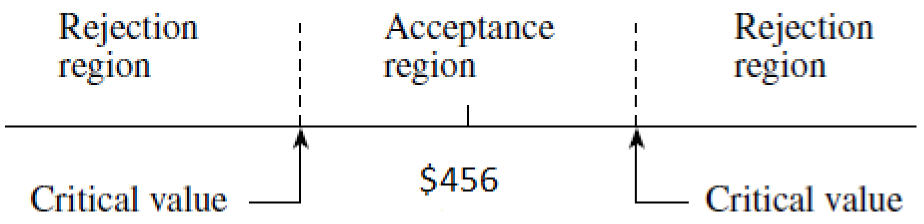
A large-sample test about a population mean

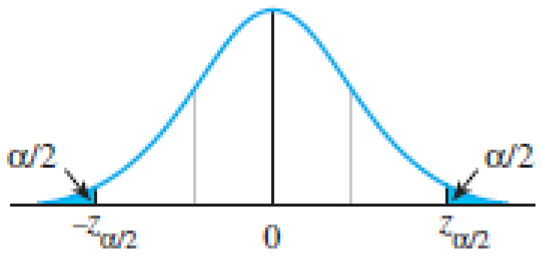
Example – The average monthly income of people in HCMC is $456. A random sample of n=51 IT professionals in HCMC showed that average income $\bar{x} = \$500$, with standard deviation $s = \$155$. Do IT professionals have monthly income **different to** the city average? Test the hypothesis with significance level $\alpha = .05$ (or 5%).

- (1) The **null hypothesis, H_0**: $\mu = \$456$
- (2) The **alternative hypothesis, H_a**: $\mu \neq \$456$

A large-sample test about a population mean

- (3) **Test statistic** – We use the same reasoning as before and come up with the test statistic z = 2.03
- (4) **Rejection region** – In 2 tailed test using significance level $\alpha = .05$, the critical values separating the rejection region and the acceptance region corresponds to $\alpha/2 = .025$ to the right and left of the tail of the standardized normal distribution. These values are $z = \pm 1.96$. The rejection region includes z < -1.96 of z > 1.96.





- (5) **Conclusion** – Because z = 2.03 is larger than 1.96, we ignore the null hypothesis and conclude that the average monthly income of IT professionals *is different to the city average*. The probability of making the wrong decision is $\alpha = 5\%$.

A large-sample test about a population mean

In summary:

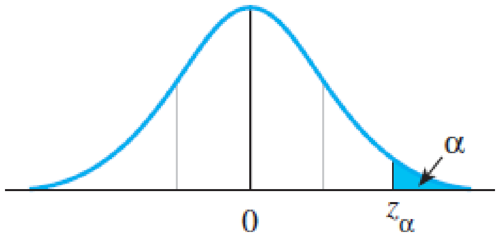
1. Null hypothesis: $H_0 : \mu = \mu_0$
2. Alternative hypothesis:

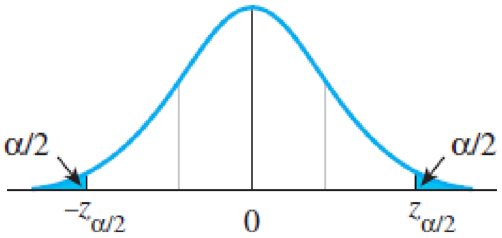
One-Tailed Test	**Two-Tailed Test**
$H_a : \mu > \mu_0$ (or, $H_a : \mu < \mu_0$)	$H_a : \mu \neq \mu_0$

3. Test statistic: $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ estimated as $z = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

4. Rejection region: Reject H_0 when

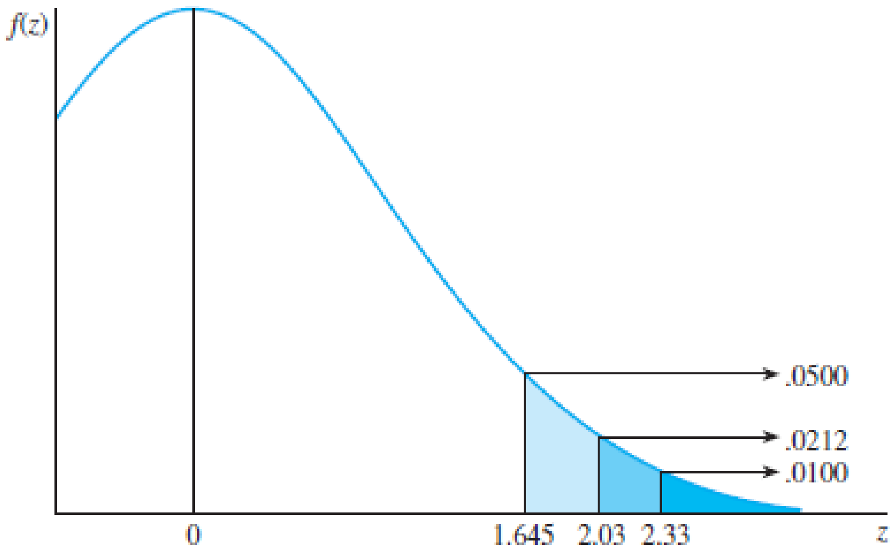
One-Tailed Test	**Two-Tailed Test**
$z > z_\alpha$ (or $z < -z_\alpha$ when the alternative hypothesis is $H_a : \mu < \mu_0$)	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$





A Large-sample test about a population mean

- In previous examples, the decision to reject a null hypothesis was based on value of z determined from a significance level α.
- In Example 1, $\alpha = .05$, the critical value of z is 1.64. We rejected the null hypothesis because the observed value of z_0 = 2.03 is larger the critical value.
- However if $\alpha = .01$, the critical value of z is 2.33, we do not reject the null hypothesis because z_0 = 2.03 is smaller the critical value. (The conclusion in this case is that the *average monthly income of IT professionals is not higher than the city average*)



A Large-sample test about a population mean

- The smallest critical value that we can use to reject H_0 is 2.03. The probability of this reject decision being wrong is P(z>2.03) = .0212, which if the p-value for the test.
- Smaller p-value means larger z_0, which means larger distance between $\mu_0 = \$456$ and sample mean $\bar{x} = \$500$, which means higher chance of rejecting the null hypothesis.

- p-value can also be compared *directly* with the significance level α.
  - *If p-value $\leq \alpha$, we reject the null hypothesis and report that the results are* ***statistically significant*** *at level α.*

- In example 1 (one-tailed test),

  p-value = P(z>2.03) = .0212
- In example 2 (two-tailed test),

  p-value = P(z>2.03) + P(z<-2.03) = .0212 + .0212 = .0424

A large-sample test about a population mean

- In example 1, if we set the significant level $\alpha = .01$, because p-value = .0212 is larger than α, we do not reject the null hypothesis and conclude that the average monthly income of IT professionals is not higher than the city average.
- Note that we do NOT say that we *accept* the null hypothesis, i.e. we do NOT conclude that the average monthly income of IT professionals *equals* the city average.
- This is because if we choose to accept the null hypothesis, we need to know the probability of error associate with such a decision.
- ***Type II error*** *for statistical test is the error of accepting the null hypothesis when it is false and an alternative hypothesis is true, represented by a probability* β.

	Null Hypothesis	
Decision	True	False
Reject H_0	Type I error	Correct decision
Accept H_0	Correct decision	Type II error