



MICROECONOMICS 2

LECTURE 8

Decision Under Risk

Outline

Decisions under risk:

Deciding whether to go to school or work, thinking about which job offer to accept, where to buy your house, what to invest your retirement savings in...

Dual approach to analyzing risk and behavior

- **First, quantify risk statistically and mathematically**
 - **Probability, Variance, Expected Value**
- **Next, model human behavior over risky scenarios**
 - **Expected Utility Theory**
 - **Non-Expected Utility Theory**

1. Assessing Risk

2. Attitudes Toward Risk

3. Reducing Risk

4. Behavioral Economics on Decisions under Risk

1. Assessing Risk

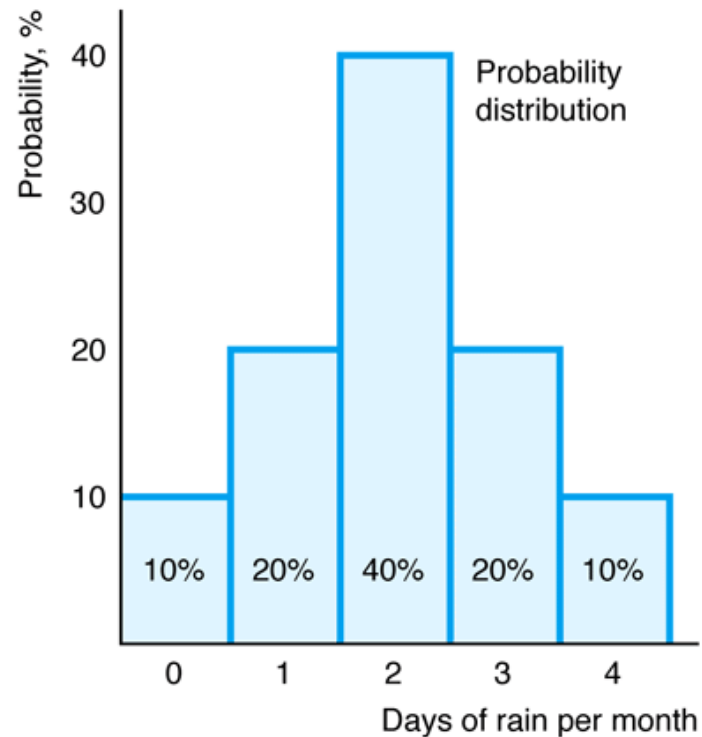
- **Risk** is the situation when the likelihood of each possible outcome is known or can be estimated, and no single possible outcome is certain to occur.
 - Ambiguity: You don't know the likelihoods; Unawareness: You don't know some possible outcomes
- A **probability** is a number between 0 and 1 that indicates the likelihood that a particular outcome will occur.
- We can estimate probability with **frequency**, the number of times that one particular outcome of an event occurred (n) out of the total number of times that the event occurred (N).
- If we don't have a history of the event that allows us to calculate frequency, we can use our best estimate or **subjective probability**.

$$\theta = \frac{n}{N}$$

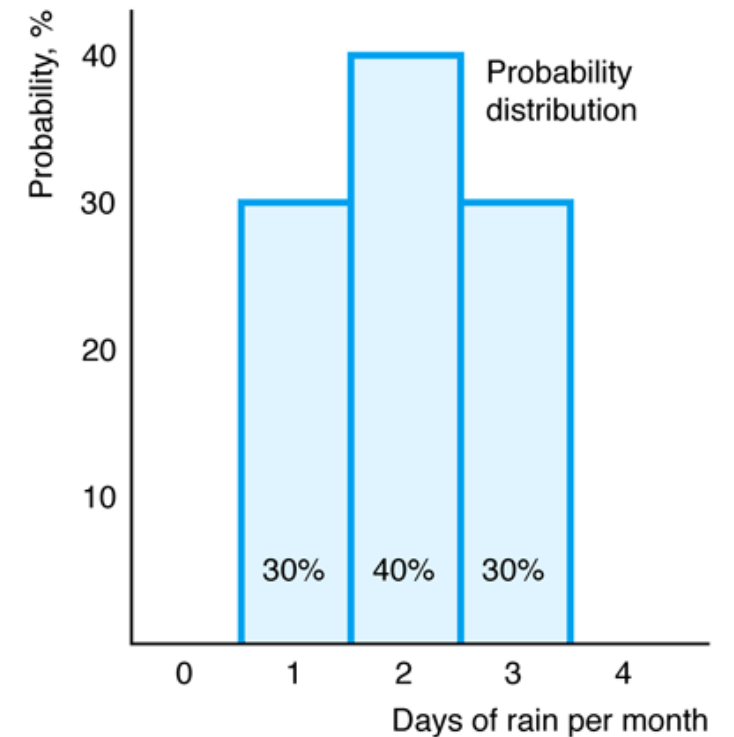
Assessing Risk

A **probability distribution** relates the probability of occurrence to each possible outcome.

(a) Less Certain



(b) More Certain



Assessing Risk

- **Expected value** is the value of each possible outcome (V_i) times the probability of that outcome (θ_i), summed over all n possible outcomes:

$$EV = \sum_{i=1}^n \theta_i V_i$$

- **Variance** measures the spread of the probability distribution or how much variation there is between the actual value and the expected value.

$$\text{Variance} = \sum_{i=1}^n \theta_i (V_i - EV)^2$$

- **Standard deviation** (σ) is the square root of the variance and is a more commonly reported measure of risk.
 - Used by economists and business people to describe risk.

Assessing Risk

- Example: Greg schedules an outdoor event. If it doesn't rain, he'll make \$15 in profit. If it does rain, he'll make -\$5 in profit (loss). There is a 50% chance of rain.

- Greg's expected value (outdoor event):
$$EV = [Pr(\text{no rain}) \times \text{Value}(\text{no rain})] + [Pr(\text{rain}) \times \text{Value}(\text{rain})]$$
$$= \left(\frac{1}{2} \times \$15\right) + \left[\frac{1}{2} \times (-\$5)\right] = \$5$$

- Variance (outdoor event):
 - Standard deviation = \$10
$$\sigma^2 = \left[\theta_1 \times (V_1 - EV)^2\right] + \left[\theta_2 \times (V_2 - EV)^2\right]$$
$$= \left[\frac{1}{2} \times (\$15 - \$5)^2\right] + \left[\frac{1}{2} \times (-\$5 - \$5)^2\right]$$
$$= \left[\frac{1}{2} \times (\$10)^2\right] + \left[\frac{1}{2} \times (-\$10)^2\right] = \$100.$$

Assessing Risk

- Example, continued: Greg schedules an indoor event. If it doesn't rain, he'll make \$10 in profit. If it does rain, he'll make \$0 in profit. There is still a 50% chance of rain.

- Greg's expected value (indoor event)... is the same!

$$EV = \left(\frac{1}{2} \times \$10\right) + \left(\frac{1}{2} \times \$0\right) = \$5$$

- Variance (indoor event)... is much smaller:

- Standard deviation = \$5

$$\begin{aligned}\sigma^2 &= \left[\frac{1}{2} \times (\$10 - \$5)^2\right] + \left[\frac{1}{2} \times (\$0 - \$5)^2\right] \\ &= \left[\frac{1}{2} \times (\$5)^2\right] + \left[\frac{1}{2} \times (-\$5)^2\right] = \$25\end{aligned}$$

- Much less risky to schedule the event indoors.

2. Attitudes Toward Risk

- Although indoor and outdoor events have the same expected value, the outdoor event involves more risk. He'll schedule the event outdoors only if he likes gambling
- People can be classified according to attitudes toward risk.
- A **fair bet** is a wager with an expected value of zero. Example: You receive \$1 if a flipped coin comes up heads and you pay \$1 if a flipped coin comes up tails.
 - Someone who is unwilling to make a fair bet is **risk averse**.
 - Someone who is indifferent about a fair bet is **risk neutral**.
 - Someone who is **risk preferring** (risk loving) will make a fair bet.
- Implication: For any lottery, a risk averse person prefers taking EV(the lottery) to taking the lottery itself: $A \text{ lottery} = EV(\text{the lottery}) + A \text{ fair bet}$
 - Vice versa for risk loving people.

Expected Utility Theory

- We can extend our model of utility maximization to include risk by assuming that people maximize expected utility.
- Expected utility, EU, is the probability-weighted average of the utility, $U(\bullet)$ from each possible outcome:

$$EU = \sum_{i=1}^n \theta_i U(V_i)$$

- The weights are the probabilities that each outcome will occur, just as in expected value.
- EU: the probability-weighted average of the **utility** from the outcomes
- EV: the probability-weighted average of the outcomes

Expected Utility Theory

- von Neumann and Morgenstern (1944) prove that a consumer whose preference satisfies completeness, transitivity, independence and continuity over any lotteries is an expected utility maximizer.
- Completeness and transitivity are defined as in Ch. 3.
- **Independence:** For any lotteries A, B, C , if $A \succsim B$, then for any $0 \leq t \leq 1$, we have $tA + (1-t)C \succsim tB + (1-t)C$.

Note: $tA + (1-t)C$ means with prob. t you get lottery A , with prob. $1-t$ you get lottery C .

- **Continuity:** For any lotteries A, B, C , if $A \succ B \succ C$, then there exists a probability p such that $B \sim pA + (1-p)C$.

Review: Consumer Theory

- To explain consumer behavior, economists assume that consumers have a set of tastes or preferences that they use to guide them in choosing between goods.
- Goods are ranked according to how much pleasure a consumer gets from consuming each
 - Preference relations summarize a consumer's ranking
 - \succ is used to convey strict preference (e.g., $a \succ b$)
 - \succsim is used to convey weak preference (e.g., $a \succsim b$)
 - \sim is used to convey indifference (e.g., $a \sim b$)

Review: Consumer Theory

Completeness

- When facing a choice between a and b , a consumer can rank them so that either $a \succ b$, $b \succ a$, or $a \sim b$.
- Completeness also holds for weak preference:
- For any a and b , either $a \succeq b$ or $b \succeq a$.

Transitivity

- Consumers' rankings are logically consistent in the sense that if $a \succ b$ and $b \succ c$, then $a \succ c$.
- Transitivity also holds for weak preference and indifference.

Rationality = Completeness + Transitivity

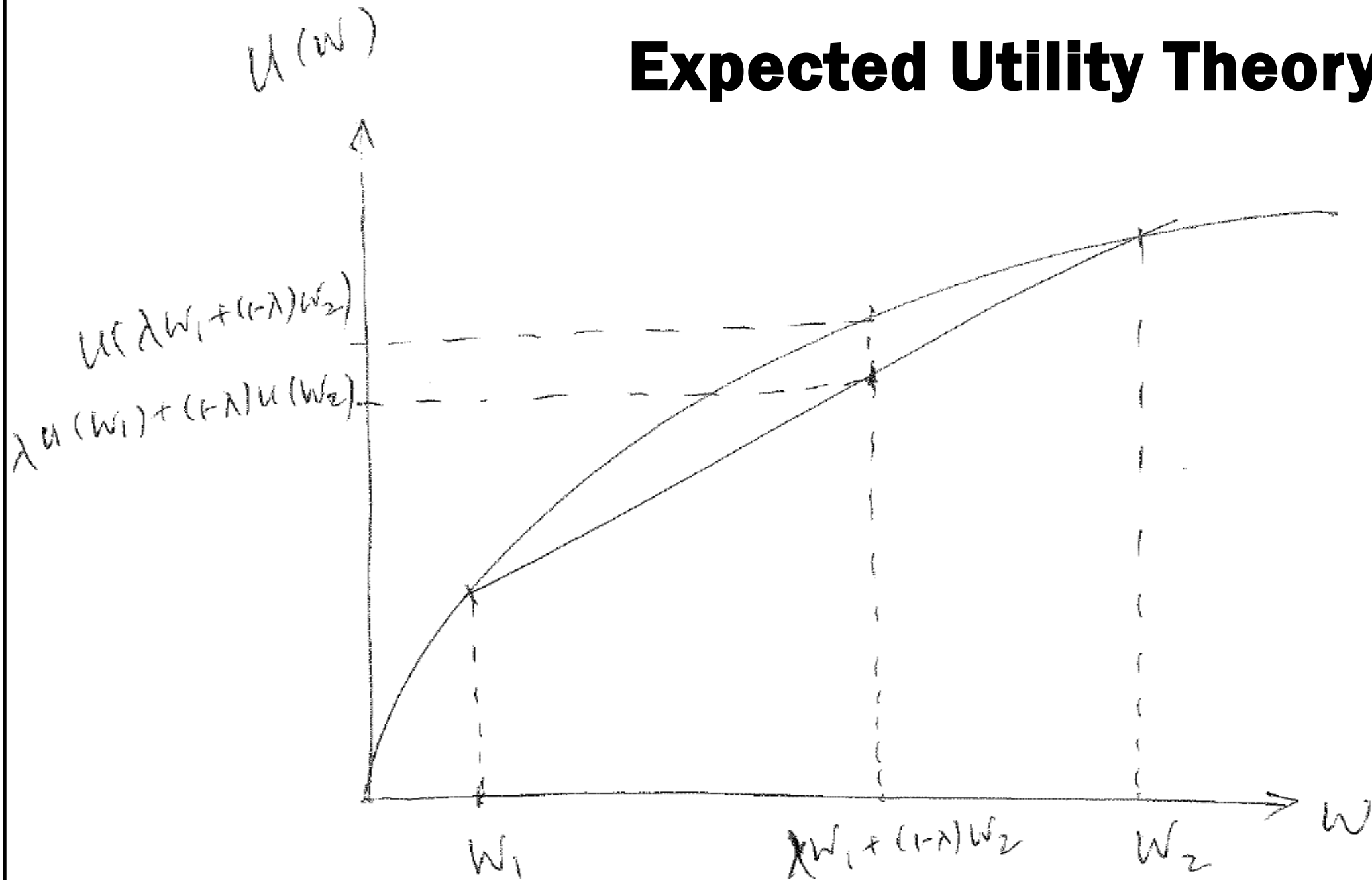
Expected Utility Theory

- With expected utility, a person whose utility function of income, $U(Y)$, is strictly concave ($U'' < 0$) is risk averse:
- Why strict concavity implies risk aversion? Mathematics
- For a strictly concave function $U(\cdot)$, we have: for any W_1 , W_2 , and any $0 < \lambda < 1$

$$U(\lambda W_1 + (1 - \lambda)W_2) > \lambda U(W_1) + (1 - \lambda)U(W_2)$$

- This means that for a gamble that gives you W_1 with prob. λ and W_2 with prob. $1 - \lambda$, you prefer receiving EV to taking the gamble.

Expected Utility Theory



Expected Utility Theory

- Why strict concavity implies risk aversion? Intuition
- Diminishing MU ($U'' < 0$): “More money is good... but \$1 more when I have \$1,000,000 is not as good as \$1 more when I have \$10!”
 - Dislikes losing more than likes winning.
 - Given the same amount of money, the loss in utility from losing it is greater than the increase in utility from winning it.

Attitudes Toward Risk

- Example (Risk-averse Irma and wealth): Irma has initial wealth of \$40
 - Option 1: keep the \$40 and do nothing $\rightarrow U(\$40) = 120$
 - Option 2: buy a vase that she thinks is a genuine Ming vase with probability of 50%
 - ✓ If she is correct, wealth = \$70 $\rightarrow U(\$70) = 140$
 - ✓ If she is wrong, wealth = \$10 $\rightarrow U(\$10) = 70$
 - ✓ Expected value of wealth remains \$40 = $(\frac{1}{2} \cdot \$10) + (\frac{1}{2} \cdot \$70)$
 - ✓ Expected value of utility is 105 = $(\frac{1}{2} \cdot 70) + (\frac{1}{2} \cdot 140)$
- Although both options have the same expected value of wealth, the option with risk has lower expected utility.

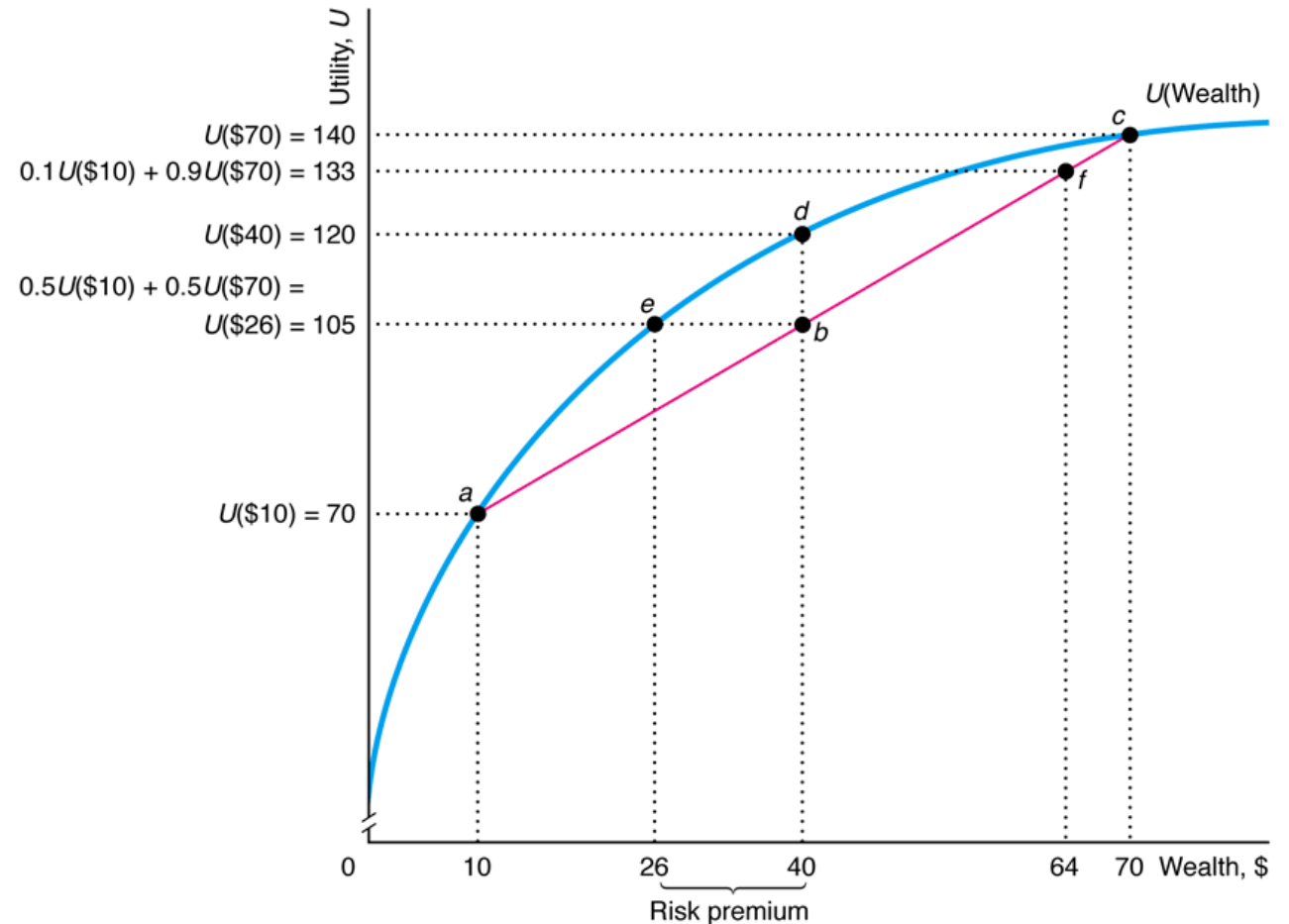
Risk Aversion

She is risk-averse and would pay a **risk premium (14)** to avoid risk.

- Risk premium is the largest amount that a risk-averse person is willing to pay to eliminate the risk and get the expected value with certainty.

26 is the **certainty equivalence** (expected value – risk premium) of Option 2:

$$U(\text{CE of Option 2}) = EU(\text{Option 2})$$

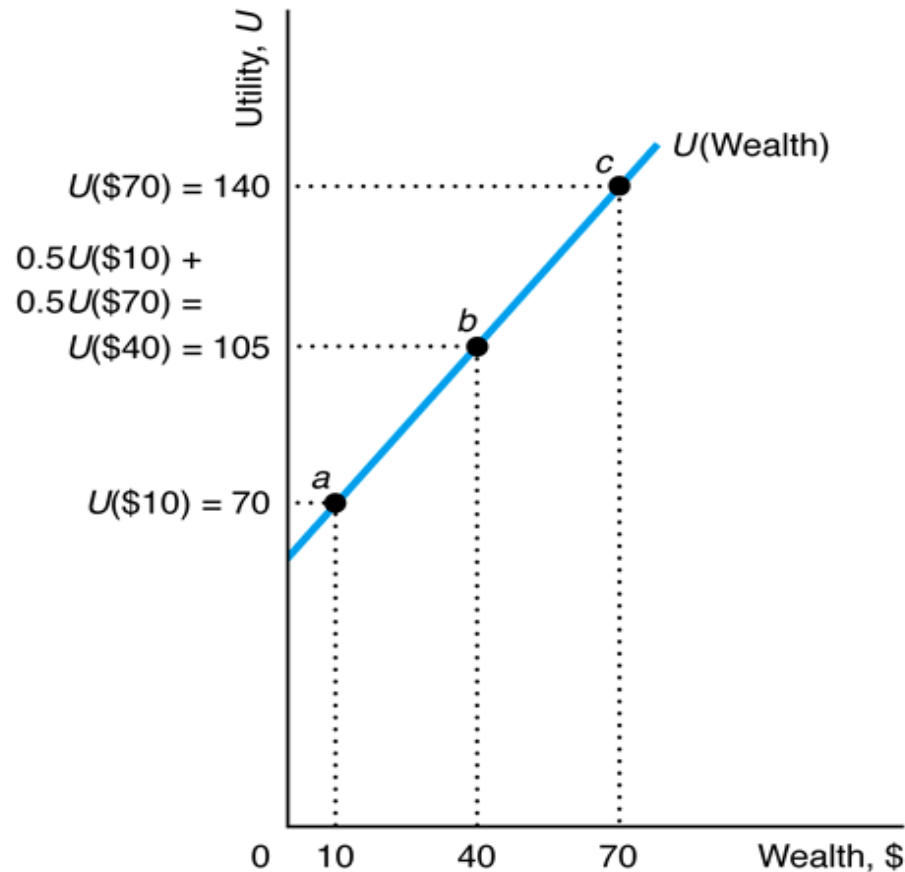


Risk Aversion

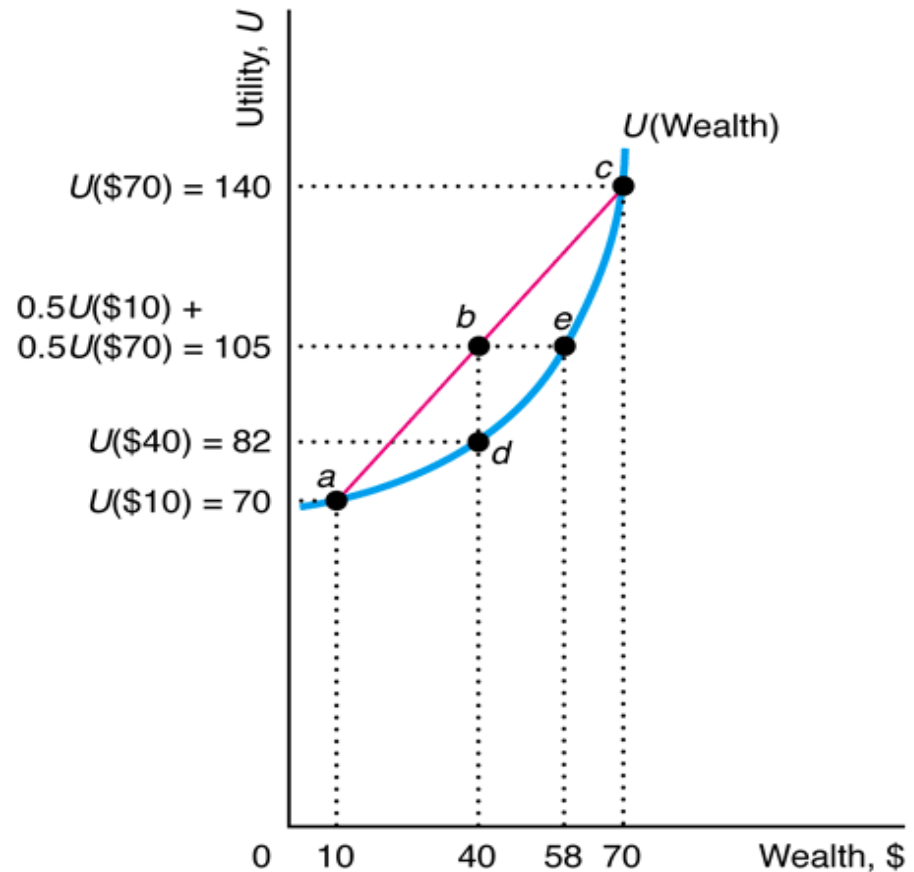
- Risk premium is the largest amount that a risk-averse person is willing to pay to avoid the risk and get the expected value for certain.
- Certainty equivalence (CE) of a risky option is defined as:
 - $U(\text{CE of the option}) = EU(\text{the option})$,
 - meaning that the utility of receiving CE for certain is equal to the expected utility of the risky option.
 - $\text{CE} = \text{EV} - \text{Risk Premium}$.

Risk Neutrality and Risk Preference

(a) Risk-Neutral Individual



(b) Risk-Preferring Individual



Degree of Risk Aversion

- The degree of risk aversion is judged by the shape of the utility function over wealth, $U(W)$.
- One common measure is the Arrow-Pratt measure of risk aversion:
$$\rho(W) = -\frac{d^2U(W)/dW^2}{dU(W)/dW}$$
- This measure is positive for risk-averse individuals, zero for risk-neutral individuals, and negative for those who prefer risk.
- There is a typo in Equation (16.4) of the textbook.

3. Reducing Risk

There are four primary ways for individuals to avoid risk:

- **Just say no:** Abstaining from risky activities is the simplest way to avoid risk.
- **Obtain information:** Armed with information, people may avoid making a risky choice or take actions to reduce probability of a disaster.
- **Diversify:** “Don’t put all your eggs in one basket.”
- **Insure:** Insurance is like paying a risk premium to avoid risk.

Avoiding Risk Via Diversification

Example: Two firms. Each firm has half chance of being worth \$40, and half chance of being worth \$10. The values of the firms are independent.

Option 1: Buy 2 shares from one firm

$$EV = 0.5 \times 80 + 0.5 \times 20 = 50$$

$$\text{Variance: } 0.5 \times (80-50)^2 + 0.5 \times (20-50)^2 = 900$$

Option 2: Buy 1 share from each firm, $\frac{1}{4}$ chance of both worth \$40, $\frac{1}{4}$ chance of both worth \$10, $\frac{1}{2}$ chance of one worth \$40 with the other worth \$10

$$EV = 0.25 \times 80 + 0.5 \times 50 + 0.25 \times 20 = 50$$

$$\text{Variance: } 0.25 \times (80-50)^2 + 0.5 \times (50-50)^2 + 0.25 \times (20-50)^2 = 450,$$

which is lower.

Avoiding Risk Via Diversification

- Diversification can reduce risk if two events are independent (uncorrelated).
- Diversification can eliminate risk if two events are perfectly negatively correlated.
 - If one event occurs, then the other won't occur.
- Diversification does not reduce risk if two events are perfectly positively correlated.
 - If one event occurs, then the other will occur, too.
- Example: investors reduce risk by buying shares in a mutual fund, which is comprised of shares of many companies.

Avoiding Risk Via Insurance

- A risk-averse individual will **fully insure** by buying enough insurance to eliminate risk if the insurance company offers **fair insurance**, which eliminates risk but **does not change expected income of the individual**.
 - In this scenario, the expected value of the insurance is zero; the policyholder's expected value with and without the insurance is the same.
- Insurance companies never offer fair insurance, because they would not stay in business, so most people do not fully insure.

4. Behavioral Economics on Risk and Uncertainty

Many individuals make choices under uncertainty that are inconsistent with expected utility theory.

1. Difficulty assessing probabilities
=> Difficulty assessing expected utility

- Gambler's fallacy
- Overconfidence

2. Behavior varies with circumstances
=> Factors other than expected utility affect decision

- Ambiguity aversion
- Framing effect
- Certainty effect (Allais paradox)

3. Prospect theory

Difficulty of Assessing Probabilities

- People often have mistaken beliefs about the probability that an event will occur.
- The **gambler's fallacy** arises from the false belief that past events affect current, independent outcomes.
 - Example: flipping 'heads' 10 times in a row does not change the probability of getting 'heads' on the next flip from 50%.
- Some people engage in risky gambles because they are overconfident.
 - Surveys of gamblers reveal a big gap between estimated chance of winning a bet and objective probability of winning.

Ambiguity

- Two urns, each with 100 red and black balls. In urn A, there are 50 red balls and 50 black balls. In urn B, the composition is unknown.
- Bet 1: a red ball will be drawn from urn A.
- Bet 2: a red ball will be drawn from urn B.
- Which bet do you prefer?
- Most would agree that the subjective probability of drawing a red ball from urn B is 50%.
- But experiments find that more prefer Bet 1.
- Ambiguity aversion: People dislike ambiguity.

Allais Paradox

- First, a group of subjects were asked to choose between two options:
 - **Option A:** receive \$4,000 with prob. 80%, and 0 with prob. 20%.
 - **Option B:** receive \$3,000 with certainty
- Which would you choose?
- 80% of experimental subjects choose B, the certain outcome.

Allais Paradox

- Then, the subjects were given another set of options:
 - Option C: receive \$4,000 with prob. 20%, and 0 with prob. 80%.
 - Option D: receive \$3,000 with prob. 25%, and 0 with prob. 75%.
- Which would you choose?
- 65% of experimental subjects prefer C.

Allais Paradox

The above behavior violates the expected utility theory.

- By expected utility theory, choosing B over A means

$$U(3000) > 0.8U(4000) + 0.2U(0)$$

$$U(3000) - U(0) > 0.8[U(4000) - U(0)] \quad (1)$$

- Choosing C over D implies $0.2U(4000) + 0.8U(0) > 0.25U(3000) + 0.75U(0)$

$$\rightarrow 0.2[U(4000) - U(0)] + U(0) > 0.25[U(3000) - U(0)] + U(0)$$

$$\rightarrow 0.2[U(4000) - U(0)] > 0.25[U(3000) - U(0)]$$

$$\rightarrow 0.8[U(4000) - U(0)] > U(3000) - U(0) \quad (2)$$

- **Contradiction** between (1) and (2).

Allais Paradox

- The above behaviour violates **independence**.
- $C = 0.25*A + 0.75*0$; $D = 0.25*B + 0.75*0$, where 0 means that receiving 0 with certainty.
 - Option A: 4000 with 80%, and 0 with 20%.
 - Option B: 3000 with certainty
 - Option C: 4000 with 20%, and 0 with 80%.
 - Option D: 3000 with 25%, and 0 with 75%.
- Independence implies that: if B is better than A, then D is better than C.
- The Allais paradox may come from the certainty effect.
- Many people put excessive weight on outcomes they consider to be certain relative to risky outcomes (certainty effect).

Kahneman and Tversky 1981 (Framing effect)

- An unusual disease is expected to kill 600 people. The government is considering two programs, A and B, to combat the disease.
- If A is adopted, 200 people will be saved.
- If B is adopted, with 1/3 prob., 600 people will be saved; with 2/3 prob., no one will be saved.
- Which would you choose?
- In KT's experiment, 72% opted for A over B.

Kahneman and Tversky 1981

- Now the two programs are C and D.
- If C is adopted, 400 people will die.
- If D is adopted, with prob. $1/3$, no one will die; with prob. $2/3$, 600 people will die.
- Which would you choose?
- In KT's experiment, 78% opted for D over C.

Kahneman and Tversky 1981

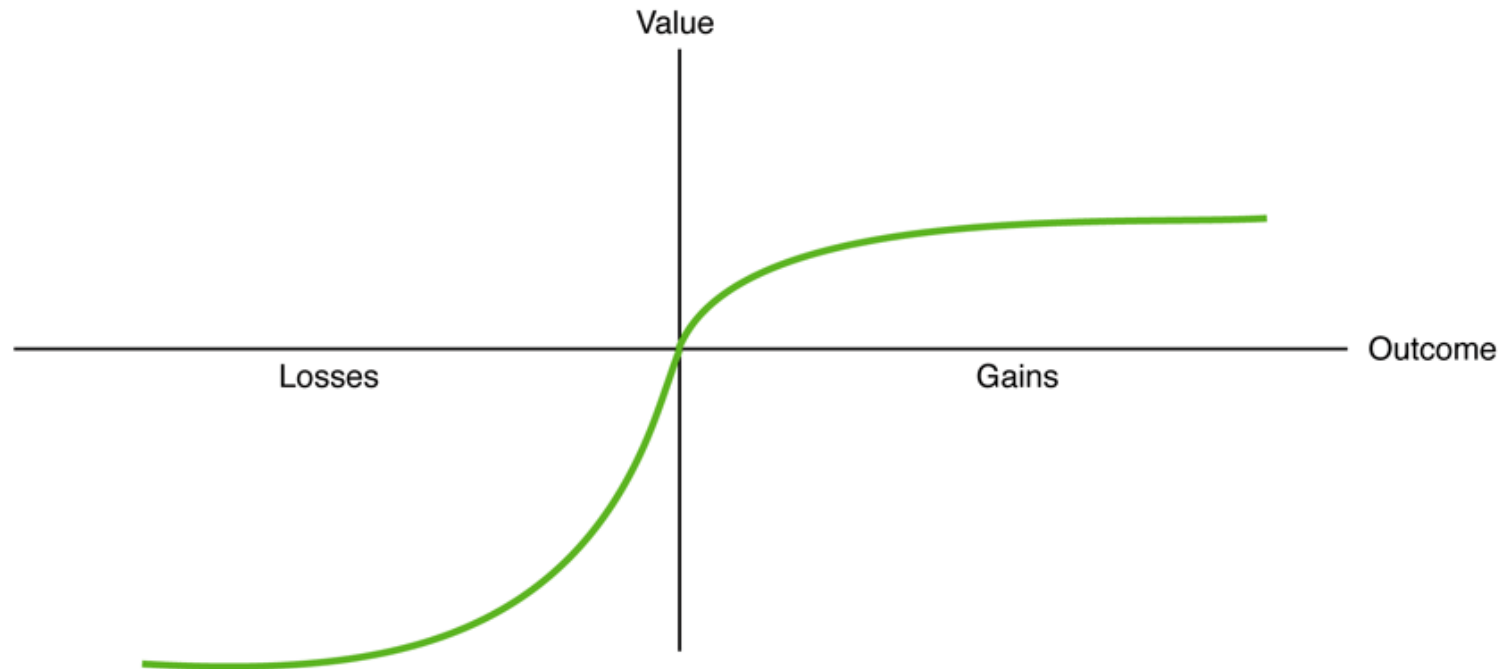
- 72% opted for A over B.
- 78% opted for D over C.
- However, A and C are equivalent, and B and D are equivalent.
- Expected utility predicts inconsistent choices for the two pairs of programs.
- Framing effect: Many people reverse their preferences when a problem is framed in a different but equivalent way.
- People are often risk averse when making choices involving gains, but often risk preferring when making choices involving losses.

Prospect Theory

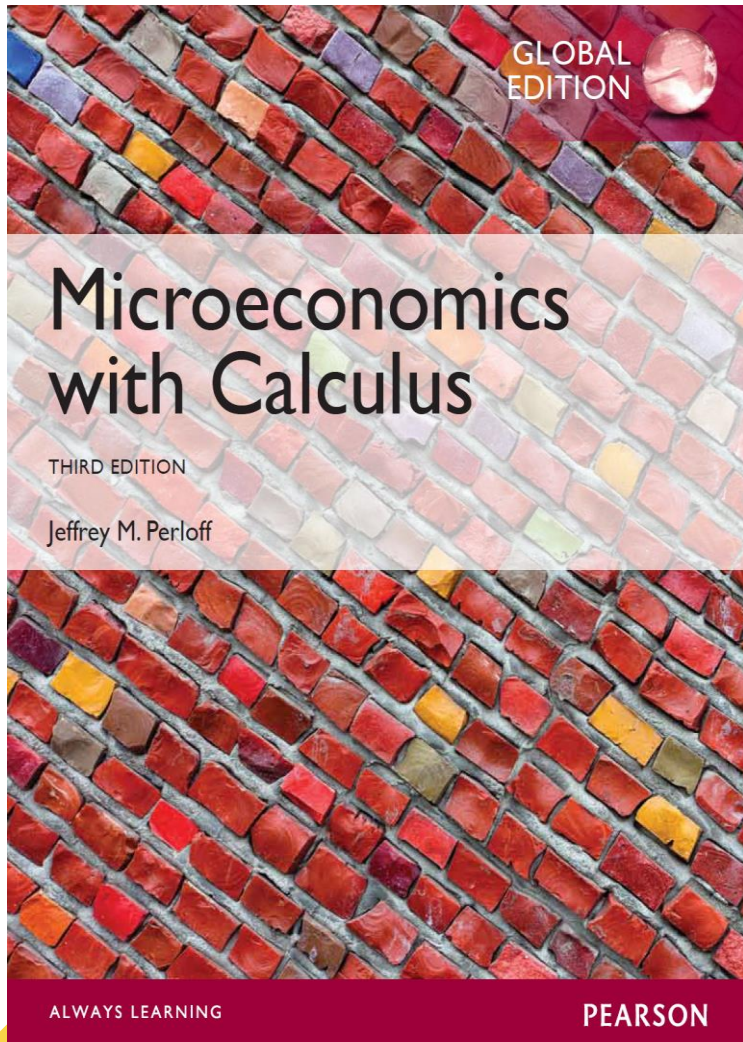
- Many theories, called Non-Expected Utility Theories, are proposed to explain one or some of the above effects. Out of these, prospect theory is most influential.
- Prospect theory: People are concerned about gains and losses in wealth (rather than the level of wealth as in expected utility theory)
- The prospect theory value function is S-shaped and has three properties:
 1. **Passes through origin:** gains/losses determined relative to initial situation (reference point)
 2. **Concave to horizontal axis:** less sensitivity to changes in large gains than small ones
 3. **Curve is asymmetric:** people treat gains and losses differently; loss aversion

Prospect Theory

- Prospect Theory Value Function



- This can explain the framing effect in the disease example.



REFERENCE

Chapter 16 - Microeconomics: Theory and Applications with Calculus, 3rd Edition. By Jeffrey M. Perloff. 2014 Pearson Education.