Microeconomics 1

Lecture 18: Game Theory

Topics for today's lecture . . .

1. The anatomy of a game
2. Sequential moves games
3. Simultaneous moves games
4. Repeated games

A flat tyre . . .

Instead of studying, two friends spend the night before an exam partying.

Turning up late to the exam, the two friends tell their professor that they are late because of a flat tyre.

The professor allows each student to take a makeup exam; the exam consists of one question: *Which tyre?*

Exercise: What if it were you?

Which tyre would you select as your answer?

(Write down your answer, we will use them again later today.)

Definition: Game theory

The study of strategic interaction; decision making when the payoff of each individual decision-maker depends on the actions of all decision-makers.

Game theory is more complex than individual optimisation as an individual's optimal action typically varies with the actions of the other players in the game.

The anatomy of a game

Players

A **player** is any party (individual or organisation) that may be faced with a strategic choice in the game.

It is typically assumed that all players in a game are:

- **Intelligent**: The players have knowledge of the structure (rules) of the game, and understand the potential consequences of their choices.
- **Rational**: Each player has complete and transitive preferences over the possible outcomes of the game, which satisfy the independence axiom.

Note: Both of these assumptions can be relaxed. However, doing so adds substantial complexity to the game, and is beyond the scope of this course.

Actions

Actions are the choices (or moves) available to a player within a game.

A game may require each player to select a single action, plan a sequence of (possibly situation-contingent) actions, or randomise between the available actions.

A plan of action that describes how a player will act in every conceivable situation is referred to as a **strategy**.

Timing

The timing of a game captures the order in which players act:

- In a sequential moves game, players take turns choosing their actions.

- In a simultaneous moves game, players take their actions at the same time.

The timing also dictates the time horizon over which players interact:

- Finite horizon games have an end.

- Infinite horizon games are repeated without end.

Payoffs

Payoffs represent the preferences of each player, over the possible outcomes of the game.

A player’s payoffs completely capture the player’s objectives within the game.

- In a properly specified game a player has no objective other than to maximise their own (expected) payoff.

If a player is an individual, the payoffs will typically be the utilities created by the outcomes of the game.

If a player is a firm, the payoff is typical the profit it receives from the game.

Discussion: Thinking about competition as a game

Competition in the market for smart-phones can be thought of as a game.

- Who do you think are the players in this game?
- What actions are available to each of these players?
- How would you describe the timing of the game?
- What are the objectives of each player, and how might you characterise their payoffs?

Sequential moves games

A market entry game

Suppose that Psi-Pharma's patent over an anti-cancer drug is about to expire.

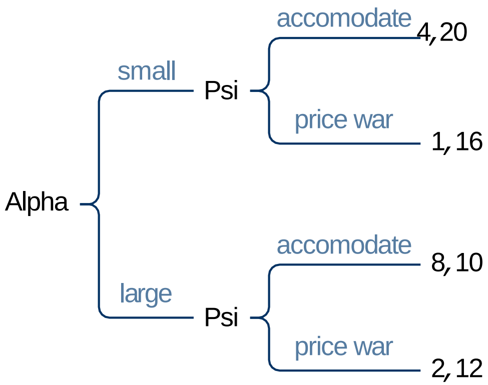
Alpha-Biotech is preparing to enter the market to compete with Psi. It has two options:

- Construct a small-capacity plant, that will have little impact on Psi.
- Construct a large-capacity plant, that will create strong competition for Psi.

Psi-Pharma can respond to Alpha's entry into the market in two ways:

- It can accomodate Alpha's entry, conceding market share.
- It can initiate a price war, aggressively discounting its price.

The game tree



The game tree illustrates the timing of the game:

- Alpha moves first, choosing the capacity of its production facilities.
- Psi then chooses how to respond.

The payoffs, in millions of dollars, are listed for each possible outcome:

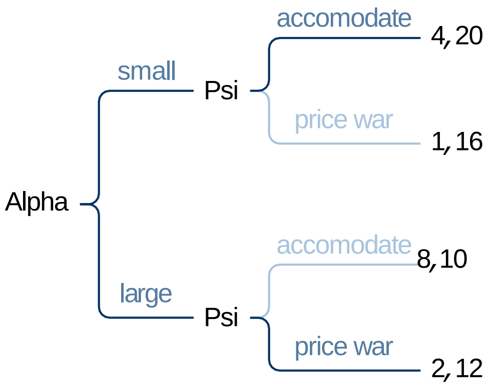
- The first number is the first-mover's (Alpha's) payoff.
- The second number is the second-mover's (Psi's) payoff.

Definition: **Best response**

The strategy (or strategies) that deliver a player the highest (expected) payoff, given the strategies of the remaining players in the game.

In order for a player to rationalise employing a strategy, it must be a best-response to the strategies that the remaining players have been observed playing, or are expected to play.

Backward induction

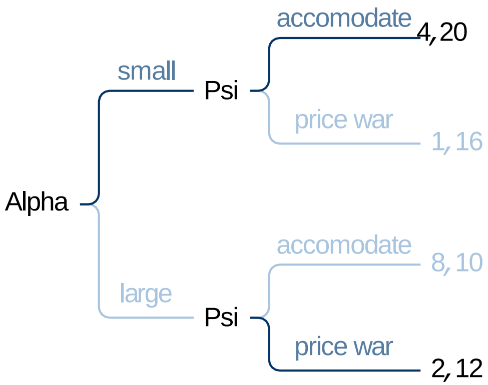


Sequential moves games can be analysed by backward induction.

Starting with the final decisions of the game, find the player's best-response to the previous actions.

- If Alpha builds a small plant, Psi's best-response is to accomodate.
- If Alpha builds a large plant, Psi's best-response is to initiate a price war.

Backward induction continued

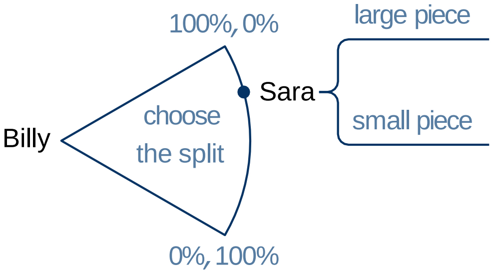


Having determined how Psi will respond to each of Alpha's actions, we see that,

- Alpha's payoff to building a small plant will be $4M.
- Alpha's payoff to building a large plant will be $2M.

Alpha Biotech's optimal entry strategy is to build a small production facility.

Exercise: The cutting the cake game



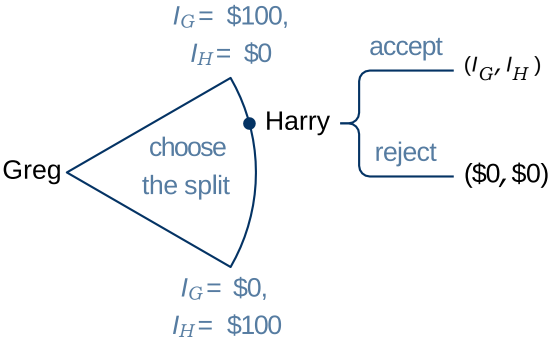
Billy and Sara are arguing over who should get the last piece of cake.

- They both think more cake is better.

Their mother tells Billy to cut the cake into two pieces, in any way he wants, however Sara will get to choose which piece she eats.

Use backward induction to solve this game.

Exercise: The ultimatum game

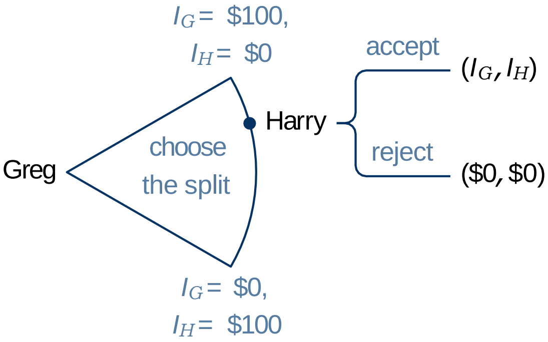


Greg and Harry are engaged in a take-it or leave-it negotiation over $100.

- Greg must propose how the $100 will be split between the two players.
- Harry can either accept the proposal, or reject it. In the event of a rejection neither player receives anything.

Use backward induction to solve this game. (Assume that the players care only about the monetary payoffs they receive.)

Discussion: The ultimatum game



What offer would you make in this game if you were Greg?

What offer would you accept in this game if you were Harry?

Is your reasoning influenced by,

- fairness?
- heuristics?
- reputation?

Simultaneous moves games

An advertising game

Suppose that Alpha-Biotech and Psi-Pharma compete in the pharmaceuticals market.

Each firm must choose an advertising strategy:

- A firm can choose a large campaign, a small campaign, or no campaign at all.
- Advertising is costly, but helps a firm to capture market share.

The two firms must select their strategies simultaneously, and without knowing what their rival is doing.

The payoff matrix

		Psi		
		Large	Small	None
Alpha	Large	0 0	8 6	9 9
	Small	12 4	16 8	15 10
	None	18 5	20 7	18 9

The payoff matrix illustrates how each possible outcome of the game affects the two players.

- Alpha’s choice of strategy determines the row.
- Psi’s choice of Strategy determines the column.
- The corresponding payoffs (in millions of dollars) can be found at the intersection of these strategies.

Definition: Nash equilibrium

A situation in which each player in chooses their best response, given the strategies chosen by the other players.

A Nash equilibrium describes play in a game that is stable in the sense that no player can improve their (expected) payoff by altering their strategy.

Finding Alpha's best responses

		Psi		
		Large	Small	None
Alpha	Large	0 / 0	8 / 6	9 / 9
	Small	12 / 4	16 / 8	15 / 10
	None	18 / 5	20 / 7	18 / 9

The first step in identifying a Nash equilibrium is to find each firm's best responses.

- If Psi chooses a large campaign, Alpha's best-response is no campaign.
- If Psi chooses a small campaign, Alpha's best-response is a small campaign.
- If Psi chooses a no campaign, Alpha's best-response is a small campaign.

Finding Phi’s best responses

		Psi		
		Large	Small	None
Alpha	Large	0, 0	6, 8	9, 9
	Small	4, 12	8, 16	10, 15
	None	5, 18	7, 20	9, 18

- If Alpha chooses a large campaign, Psi’s best-response is no campaign.
- If Alpha chooses a small campaign, Psi’s best-response is a small campaign.
- If Alpha chooses a no campaign, Psi’s best-response is a small campaign.

The pure-strategy Nash equilibrium is for each firm to choose a small campaign, as this is the mutual best response.

The game of chicken

		Sara	
		Swerve	Straight
Billy	Swerve	0, 0	2, -2
	Straight	-2, 2	-10, -10

Billy and Sara are playing a game of chicken; riding their bikes towards each other.

- Whoever swerves loses.
- Crashing is even worse.

Both players’ best-responses are to do the opposite of her/his rival.

There are two pure-strategy Nash equilibria to this game.

Exercise: What if it were you? (continued)

Recall the story of the two friends who were late for an exam.

In the makeup exam, each friend can choose between: Front left, front right, rear left, and rear right.

1. If the two friends take the exam at the same time, and cannot communicate, what are the possible pure-strategy equilibria of the game? (You do not need to construct a payoff matrix.)

2. Compare the answer you wrote down earlier with your neighbours. Did your choices constitute an equilibrium?

Rock, paper, scissors

		Sara		
		Rock	Paper	Scissors
Billy	Rock	0 0	1 -1	-1 1
	Paper	-1 1	0 0	1 -1
	Scissors	1 -1	-1 1	0 0

This is the payoff matrix for the game Rock, Paper, Scissors.

Each player's best-response is to choose the strategy that defeats her/his rival's choice.

There is no cell in which the two players' best-responses intersect.

- If a player is predictable she/he loses.
- The solution is to be unpredictable.

Definition: Mixed strategy

A probability distribution by which a player randomly selects a pure strategy.

Every game that can be characterised by a payoff matrix has at least one (possibly mixed-strategy) Nash equilibrium.

Mixed strategy Nash equilibrium

		Sara		
		Rock	Paper	Scissors
	Rock	0 0	1 -1	-1 1
Billy	Paper	-1 1	0 0	1 -1
	Scissors	1 -1	-1 1	0 0

If Billy (or Sara) selects each pure strategy with probability 1/3, Sara (or Billy) can do no better than to pick a strategy at random.

It follows that each player choosing each pure strategy with probability 1/3, is a mixed strategy Nash equilibrium.

Quiz 1

		Sara		
		Left	Centre	Right
Billy	Up	10, 20	9, 32	3, 40
	Middle	5, 5	1, 1	8, 6
	Down	12, 35	7, 40	5, 40

Sara's best-response to Billy playing Down is,

(a) Left.

(b) Centre.

(c) Right.

(d) Both Centre and Right.

Quiz 2

		Sara		
		Left	Centre	Right
	Up	20 10	32 9	40 3
Billy	Middle	5 5	1 1	6 8
	Down	35 12	40 7	40 5

How many pure-strategy Nash equilibria does this game possess?

(a) 0.

(b) 1.

(c) 2.

(d) 3.

Repeated games

Household chores

		Sara	
		Clean	Shirk
Billy	Clean	15 / 15	5 / 20
	Shirk	20 / 5	10 / 10

Sara and Billy both have chores to do around the house.

- Cleaning the house makes it more pleasant for them both.
- Housework is tedious and they would each rather be doing something else.

Billy's best-responses are to shirk.

Sara's best-responses are likewise to shirk.

Definition: Dominant strategy

A strategy that is a best-response regardless of which strategies are employed by the remaining players.

If all players have a dominant strategy, every player selecting their dominant strategy is a Nash equilibrium.

Prisoners’ dilemma

The Prisoner’s Dilemma

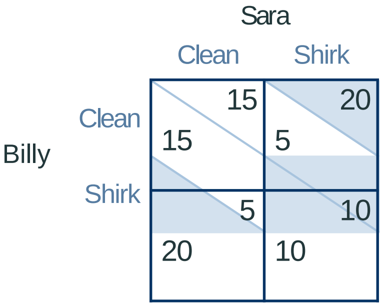
	B stays silent (cooperates)	B betrays A (defects)
A stays silent (cooperates)	Both serve 1 year	A serves 3 years, B goes free
A betrays B (defects)	A goes free, B serves 3 years	Both serve 2 years

Question:

What is the dominant strategy for each prisoner?

Source: greenflux.com

Prisoners' dilemma



This is an example of a prisoners' dilemma:

- A game in which each player has a dominant strategy.
- But in which the Nash equilibrium delivers the worst possible collective outcome.

The outcome of the game would be improved if Billy and Sara could find a way to cooperate.

Cooperation in games

In many games, cooperation will require individual players to act against their self interest (choose a strategy that is not a best response), in order to maximise the collective welfare of all players.

Cooperation can be facilitated in repeated games if the value of the future relationship is sufficient to motivate players to take cooperative actions today.

- Cheating on the agreement increases a players payoff today.
- But results in the player losing the benefits of cooperation in the future.

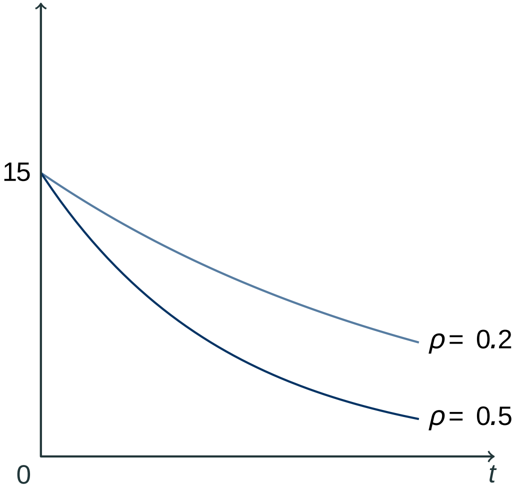
Cooperation and cheating

		Sara	
		Clean	Shirk
Billy	Clean	15 15	20 5
	Shirk	5 20	10 10

Suppose that Billy and Sara play this game every week.

- If both players do their share of the cleaning, they both receive a payoff of 15.
- If Billy shirks and doesn't do his share of the cleaning, then his payoff is 20.
- If neither player cleans, they both receive a payoff of 10.

Advanced: Discounting the future



Decision-makers typically discount future payoffs.

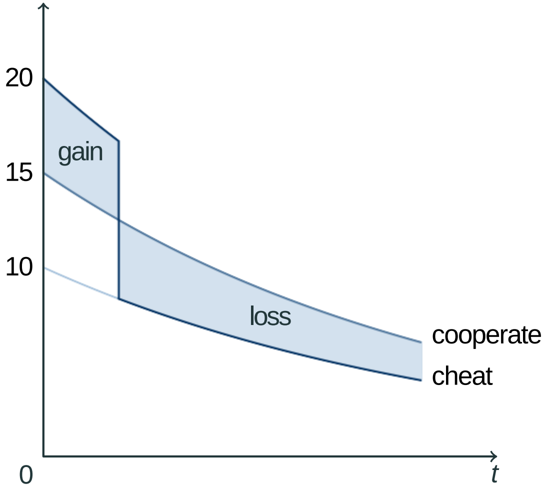
Suppose that a decision-maker discounts the future at a rate $\rho > 0$.

A payoff received at a time t in the future will be discounted by the factor,

$$\frac{1}{(1+\rho)^t}$$

Note: You will not be asked to calculate discounted payoffs in this course.

Grim trigger strategy

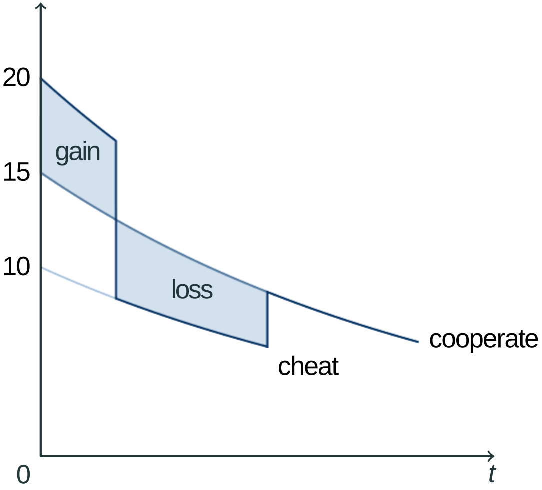


The grim trigger strategy uses the following rule:

- Cooperate so as long as the other player does the same.
- If either player ever cheats, choose the dominant strategy thereafter.

This is an equilibrium so long as the once-off gain from cheating is less than the discounted cost of lost future cooperation.

Temporary punishments

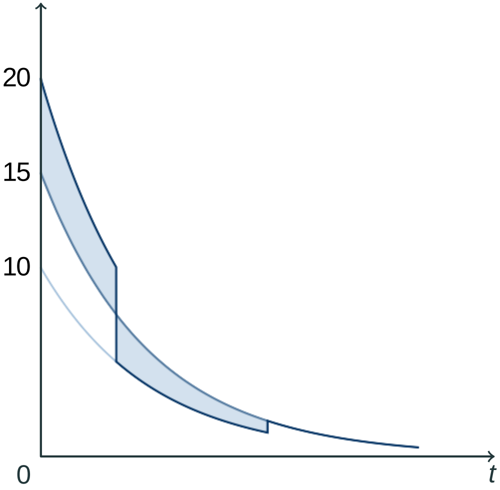


Under the grim trigger strategy a single mistake ends cooperation forever.

An alternative is temporary punishments:

- Cooperate so as long as the other player does the same.
- If either player cheats, choose the dominant strategy for the specified punishment period.
- After a punishment ends, restore cooperation.

Factors that undermine cooperation



- Players discount the future heavily (they are impatient).
- Players interact infrequently.
- Cheating is hard to detect.
- The one-time gain from cheating is large relative to the gains from cooperation.

Exercise: Unravelling

		Sara	
		Clean	Shirk
Billy	Clean	15 15	20 5
	Shirk	5 20	10 10

Suppose that Sara and Billy know they will play this game twice.

1. What action will each player choose the second time they play?

2. What action will each player choose the first time they play?

3. Now suppose that Billy and Sara know that the game will be repeated 20 times. What will be the outcome of this game?

The prospect of future interaction

In order to sustain cooperation, there must be, at every point in the game, the prospect for future interaction.

- This does not mean that the game must continue infinitely.
- Rather, at every point in time there must be some possibility that the game will be played again in the future.

If the game has a known end, then cooperation unravels in the manner of the previous exercise.

Questions?

Key concepts from today’s lecture

You can use these concepts (as search terms) to conduct further research into the topics covered in today’s lecture:

- Game theory
- Sequential moves
- Best response
- Backward induction
- First/second mover advantage
- Simultaneous moves
- Nash equilibrium
- Mixed strategy
- Repeated games
- Dominant strategy
- Cooperation & cheating
- Unravelling

Reading

Chapter 14 in Microeconomics 5th edition, by Besanko and Braeutigam.