Game Theory for Swingers

What states should the candidates visit before Election Day?

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Some campaign decisions are easy, even near the finish of a deadlocked race. Bush won't be making campaign stops in Maryland, and Kerry won't be running ads in Montana. The hot venues are Florida, Ohio, and Pennsylvania, which have in common rich caches of electoral votes and a coquettish reluctance to settle on one of their increasingly fervent suitors. Unsurprisingly, these states have been the three most frequent stops for both candidates.

Conventional wisdom says Kerry can't win without Pennsylvania, which suggests he should concentrate all his energy there. But doing that would leave Florida and Ohio undefended and make it easier for Bush to win both. Maybe Kerry should foray into Ohio too, which might lead Bush to try to pick off Pennsylvania, which might divert his campaign's energy from Florida just enough for Kerry to snatch it away. ... You see the difficulty: As in any tactical problem, the best thing for Kerry to do depends on what Bush does, and the best thing for Bush to do depends on what Kerry does. At times like this, the division of mathematics that comes to our aid is game theory.

To simplify our problem, let's suppose it's the weekend before Election Day and each candidate can only schedule one more visit. We'll concede Pennsylvania to Kerry; then for Bush to win the election, he must win both Florida and Ohio. Let's say that Bush has a 30 percent chance of winning Ohio and a 70 percent chance at Florida. Furthermore, we'll assume that Bush can increase his chances by 10 percent in either state by making a last-minute visit there, and that Kerry can do the same.

If Bush and Kerry both visit the same state, then Bush's chances remain 30 percent in Ohio and 70 percent in Florida, and his chance of winning the election is 0.3 x 0.7, or 21 percent. If Bush visits Ohio and Kerry goes to Florida, Bush has a 40 percent chance in Ohio and a 60 percent chance in Florida, giving him a 0.4 x 0.6, or 24 percent chance of an overall win. Finally, if Bush visits Florida and Kerry visits Ohio, Bush's chances are 20 percent and 80 percent, and his chance of winning drops to 16 percent.

What Bush's advisers ought to notice here is that, whatever Kerry does, Bush is better off if he visits Ohio! Visiting Ohio is what game theorists call a **dominant strategy**, and it makes game theory pretty easy: Bush should go to Ohio and ignore Kerry. If you run the numbers, you'll find that going to Ohio is a dominant strategy for Kerry, too, which means that if both campaigns act rationally they'll converge somewhere near Dayton and cancel each other out.

The combination of the Bush and Kerry strategies is an example of a **Nash equilibrium**. In general, we say that a game between two players B and K is in Nash equilibrium under the following condition: *B and K would each be satisfied with their current strategy, even if they knew in advance what their opponent's strategy would be*.

Now, let's change the game. Suppose Bush starts out with a 50-50 chance in each state. If Bush and Kerry visit the same state, Bush's chance is 0.5 x 0.5 = 0.25. If they go different ways, Bush's chance is 0.6 x 0.4 = 0.24. In other words, Bush prefers to visit the same state as Kerry, and Kerry prefers the opposite. It seems there's no possibility of a Nash equilibrium here—whatever strategies the two candidates choose, either Bush or Kerry will want to switch.

But there *is* a Nash equilibrium; it's just a bit more subtle. Suppose Bush flips a coin. If it comes up heads, he goes to Florida; tails, he goes to Ohio. Kerry does the same. Is Bush happy with his strategy? Certainly—given that Kerry plans his visit randomly, it doesn't matter what Bush does. Whatever choice he makes, there's a 50-50 chance he'll wind up in the same state as Kerry, which means his chance at winning is 0.5 x 0.25 + 0.5 x .24, or .245. Since the same computation applies to Kerry, we've arrived at a Nash equilibrium. A strategy like this one, where chance plays a part in determining a player's action, is called a **mixed strategy**, and in this case it's the strategy that game theory recommends.

How can it be to either player's advantage to outsource his campaign management to a coin flip? The key is that rational behavior tends to be predictable, and in a game of strategy, predictability will leave you with a decided disadvantage. Think of rock, paper, scissors—you're doomed if your opponent can guess your next move. Or ask yourself why baseball pitchers don't just throw their best fastball time after time. Acting at random may not seem strategic, but sometimes it's the best strategy there is.

Of course, mixed strategies don't work very well unless the players act simultaneously, which is why I started with the assumption that we were on election eve. If Kerry flips his coin a week before Election Day instead, there's plenty of time for Bush to match him visit for visit in whatever state the coin chooses. But games with multiple rounds, where each player gets the chance to respond to the other's moves, pose a game-theoretic problem beyond the scope of this article.

That's not the only simplification we made in crushing a real-world strategic problem down to something math could handle. Let's now try to make the model more realistic by putting Pennsylvania back in play. How should Bush and Kerry arrange their visits to maximize their chances of winning two of the big three? If we assume that each state is equally likely to tip toward either candidate, the question is simply: How should Bush allocate his travel time so that, in two out of three states, he's made more visits than Kerry?

This is what game theorists call a Colonel Blotto game, and, once again, only mixed strategies can be Nash equilibria.

On the other hand, if the states have different profiles—say, Bush's chances of winning Florida, Ohio, and Pennsylvania are 80 percent, 60 percent, and 20 percent, respectively—then there is a dominant strategy. In this case, it's "spend your money in Ohio"—it turns out that it's a better idea to swing the state in the middle than to try to pick off Pennsylvania or shore up Florida. In fact, the "spend the money in the middle state" strategy is dominant whenever Bush's probabilities of victory in the three states are widely separated. (Math fans can check out my calculations for the three-state scenario here.)

Then again, Bush doesn't *know* the probability he'll win in Florida; all he can do is estimate this number by Bayesian inference, as I discussed two weeks ago. We also haven't taken into account Florida's 27 electoral votes, which make it a bigger prize than Pennsylvania or Ohio. Even if we did that, 47 states would still be absent from our analysis. So, don't rush to judge the candidates' real-world strategies against the math we did here; the problems they face are too hard to be hashed out in a few lines of algebra.